



Mechanics of Materials

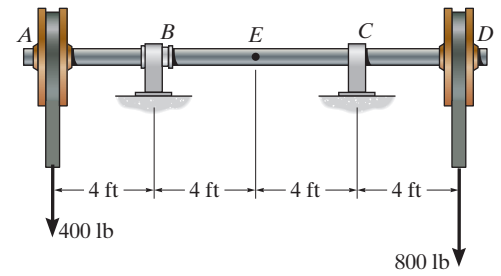
NINTH EDITION c2014

**INSTRUCTOR'S
SOLUTION
MANUAL**

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1-1. The shaft is supported by a smooth thrust bearing at B and a journal bearing at C . Determine the resultant internal loadings acting on the cross section at E .



Support Reactions: We will only need to compute C_y by writing the moment equation of equilibrium about B with reference to the free-body diagram of the entire shaft, Fig. a .

$$\zeta + \sum M_B = 0; \quad C_y(8) + 400(4) - 800(12) = 0 \quad C_y = 1000 \text{ lb}$$

Internal Loadings: Using the result for C_y , section DE of the shaft will be considered. Referring to the free-body diagram, Fig. b ,

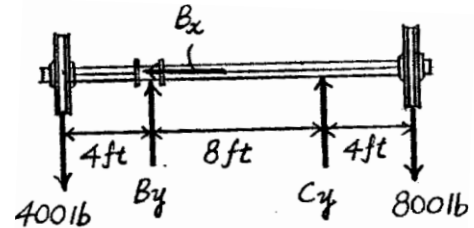
$$\rightarrow \sum F_x = 0; \quad N_E = 0 \quad \text{Ans.}$$

$$+\uparrow \sum F_y = 0; \quad V_E + 1000 - 800 = 0 \quad V_E = -200 \text{ lb} \quad \text{Ans.}$$

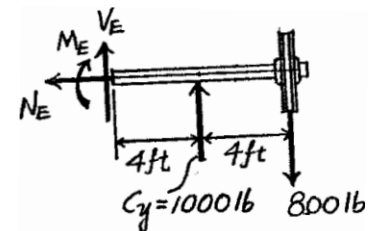
$$\zeta + \sum M_E = 0; \quad 1000(4) - 800(8) - M_E = 0$$

$$M_E = -2400 \text{ lb} \cdot \text{ft} = -2.40 \text{ kip} \cdot \text{ft} \quad \text{Ans.}$$

The negative signs indicates that V_E and M_E act in the opposite sense to that shown on the free-body diagram.



(a)

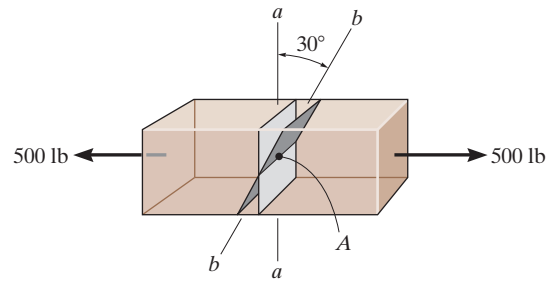


(b)

Ans:

$$N_E = 0, V_E = -200 \text{ lb}, M_E = -2.40 \text{ kip} \cdot \text{ft}$$

1-2. Determine the resultant internal normal and shear force in the member at (a) section $a-a$ and (b) section $b-b$, each of which passes through point A . The 500-lb load is applied along the centroidal axis of the member.



(a)

$$\rightarrow \Sigma F_x = 0; \quad N_a - 500 = 0$$

$$N_a = 500 \text{ lb}$$

$$+\downarrow \Sigma F_y = 0; \quad V_a = 0$$

(b)

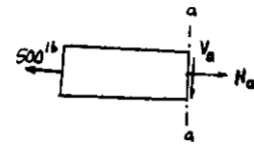
$$\swarrow \Sigma F_x = 0; \quad N_b - 500 \cos 30^\circ = 0$$

$$N_b = 433 \text{ lb}$$

$$+\nearrow \Sigma F_y = 0; \quad V_b - 500 \sin 30^\circ = 0$$

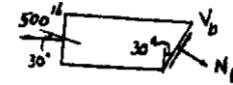
$$V_b = 250 \text{ lb}$$

Ans.



Ans.

Ans.



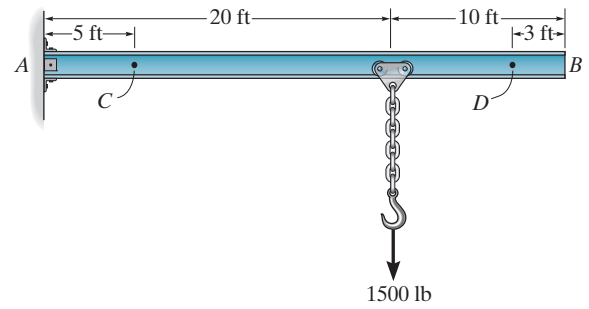
Ans.

Ans:

$$N_a = 500 \text{ lb}, V_a = 0,$$

$$N_b = 433 \text{ lb}, V_b = 250 \text{ lb}$$

1-3. The beam AB is fixed to the wall and has a uniform weight of 80 lb/ft . If the trolley supports a load of 1500 lb , determine the resultant internal loadings acting on the cross sections through points C and D .



Segment BC :

$$\leftarrow \Sigma F_x = 0; \quad N_C = 0$$

Ans.

$$+\uparrow \Sigma F_y = 0; \quad V_C - 2.0 - 1.5 = 0$$

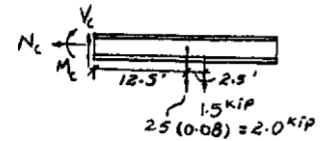
$$V_C = 3.50 \text{ kip}$$

Ans.

$$\zeta + \Sigma M_C = 0; \quad -M_C - 2(12.5) - 1.5(15) = 0$$

$$M_C = -47.5 \text{ kip} \cdot \text{ft}$$

Ans.



Segment BD :

$$\leftarrow \Sigma F_x = 0; \quad N_D = 0$$

Ans.

$$+\uparrow \Sigma F_y = 0; \quad V_D - 0.24 = 0$$

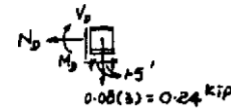
$$V_D = 0.240 \text{ kip}$$

Ans.

$$\zeta + \Sigma M_D = 0; \quad -M_D - 0.24(1.5) = 0$$

$$M_D = -0.360 \text{ kip} \cdot \text{ft}$$

Ans.

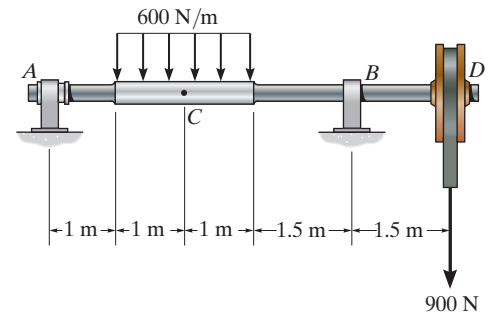


Ans:

$$N_C = 0, V_C = 3.50 \text{ kip}, M_C = -47.5 \text{ kip} \cdot \text{ft},$$

$$N_D = 0, V_D = 0.240 \text{ kip}, M_D = -0.360 \text{ kip} \cdot \text{ft}$$

*1-4. The shaft is supported by a smooth thrust bearing at A and a smooth journal bearing at B . Determine the resultant internal loadings acting on the cross section at C .



Support Reactions: We will only need to compute B_y by writing the moment equation of equilibrium about A with reference to the free-body diagram of the entire shaft, Fig. a .

$$\zeta + \Sigma M_A = 0; \quad B_y(4.5) - 600(2)(2) - 900(6) = 0 \quad B_y = 1733.33 \text{ N}$$

Internal Loadings: Using the result of B_y , section CD of the shaft will be considered. Referring to the free-body diagram of this part, Fig. b ,

$$\leftarrow \Sigma F_x = 0; \quad N_C = 0$$

Ans.

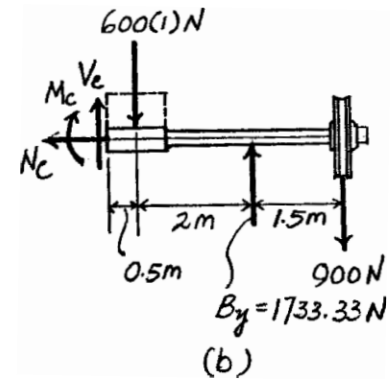
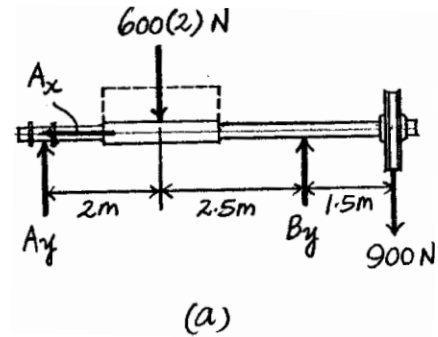
$$+\uparrow \Sigma F_y = 0; \quad V_C - 600(1) + 1733.33 - 900 = 0 \quad V_C = -233 \text{ N}$$

Ans.

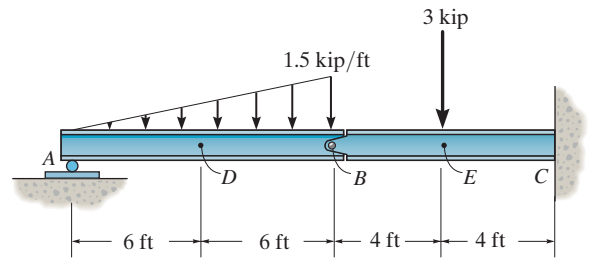
$$\zeta + \Sigma M_C = 0; \quad 1733.33(2.5) - 600(1)(0.5) - 900(4) - M_C = 0$$

$$M_C = 433 \text{ N} \cdot \text{m} \quad \text{Ans.}$$

The negative sign indicates that V_C act in the opposite sense to that shown on the free-body diagram.



1-5. Determine the resultant internal loadings in the beam at cross sections through points *D* and *E*. Point *E* is just to the right of the 3-kip load.



Support Reactions: For member *AB*

$$\zeta + \Sigma M_B = 0; \quad 9.00(4) - A_y(12) = 0 \quad A_y = 3.00 \text{ kip}$$

$$\rightarrow \Sigma F_x = 0; \quad B_x = 0$$

$$+\uparrow \Sigma F_y = 0; \quad B_y + 3.00 - 9.00 = 0 \quad B_y = 6.00 \text{ kip}$$

Equations of Equilibrium: For point *D*

$$\rightarrow \Sigma F_x = 0; \quad N_D = 0$$

Ans.

$$+\uparrow \Sigma F_y = 0; \quad 3.00 - 2.25 - V_D = 0$$

$$V_D = 0.750 \text{ kip}$$

Ans.

$$\zeta + \Sigma M_D = 0; \quad M_D + 2.25(2) - 3.00(6) = 0$$

$$M_D = 13.5 \text{ kip} \cdot \text{ft}$$

Ans.

Equations of Equilibrium: For point *E*

$$\rightarrow \Sigma F_x = 0; \quad N_E = 0$$

Ans.

$$+\uparrow \Sigma F_y = 0; \quad -6.00 - 3 - V_E = 0$$

$$V_E = -9.00 \text{ kip}$$

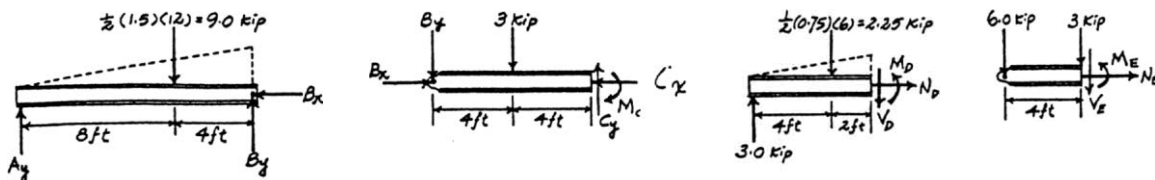
Ans.

$$\zeta + \Sigma M_E = 0; \quad M_E + 6.00(4) = 0$$

$$M_E = -24.0 \text{ kip} \cdot \text{ft}$$

Ans.

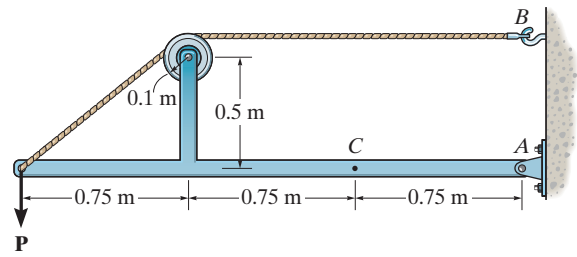
Negative signs indicate that M_E and V_E act in the opposite direction to that shown on FBD.



Ans:

$$N_D = 0, \quad V_D = 0.750 \text{ kip}, \quad M_D = 13.5 \text{ kip} \cdot \text{ft}, \\ N_E = 0, \quad V_E = -9.00 \text{ kip}, \quad M_E = -24.0 \text{ kip} \cdot \text{ft}$$

1-6. Determine the normal force, shear force, and moment at a section through point C . Take $P = 8 \text{ kN}$.

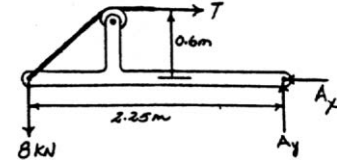


Support Reactions:

$$\zeta + \Sigma M_A = 0; \quad 8(2.25) - T(0.6) = 0 \quad T = 30.0 \text{ kN}$$

$$\rightarrow \Sigma F_x = 0; \quad 30.0 - A_x = 0 \quad A_x = 30.0 \text{ kN}$$

$$+\uparrow \Sigma F_y = 0; \quad A_y - 8 = 0 \quad A_y = 8.00 \text{ kN}$$



Equations of Equilibrium: For point C

$$\rightarrow \Sigma F_x = 0; \quad -N_C - 30.0 = 0$$

$$N_C = -30.0 \text{ kN}$$

Ans.

$$+\uparrow \Sigma F_y = 0; \quad V_C + 8.00 = 0$$

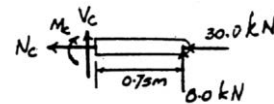
$$V_C = -8.00 \text{ kN}$$

Ans.

$$\zeta + \Sigma M_C = 0; \quad 8.00(0.75) - M_C = 0$$

$$M_C = 6.00 \text{ kN} \cdot \text{m}$$

Ans.



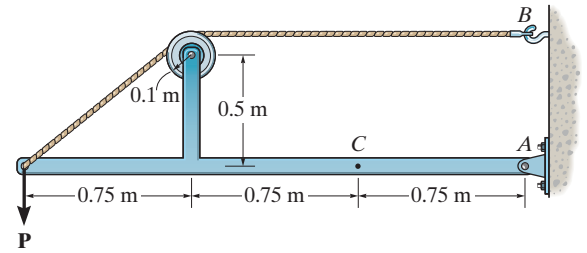
Negative signs indicate that N_C and V_C act in the opposite direction to that shown on FBD.

Ans:

$$N_C = -30.0 \text{ kN}, \quad V_C = -8.00 \text{ kN},$$

$$M_C = 6.00 \text{ kN} \cdot \text{m}$$

1-7. The cable will fail when subjected to a tension of 2 kN. Determine the largest vertical load P the frame will support and calculate the internal normal force, shear force, and moment at the cross section through point C for this loading.



Support Reactions:

$$\zeta + \Sigma M_A = 0; \quad P(2.25) - 2(0.6) = 0$$

$$P = 0.5333 \text{ kN} = 0.533 \text{ kN}$$

$$\rightarrow \Sigma F_x = 0; \quad 2 - A_x = 0 \quad A_x = 2.00 \text{ kN}$$

$$+\uparrow \Sigma F_y = 0; \quad A_y - 0.5333 = 0 \quad A_y = 0.5333 \text{ kN}$$

Equations of Equilibrium: For point C

$$\rightarrow \Sigma F_x = 0; \quad -N_C - 2.00 = 0$$

$$N_C = -2.00 \text{ kN}$$

$$+\uparrow \Sigma F_y = 0; \quad V_C + 0.5333 = 0$$

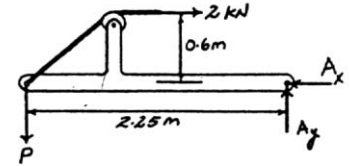
$$V_C = -0.533 \text{ kN}$$

$$\zeta + \Sigma M_C = 0; \quad 0.5333(0.75) - M_C = 0$$

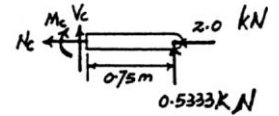
$$M_C = 0.400 \text{ kN} \cdot \text{m}$$

Negative signs indicate that N_C and V_C act in the opposite direction to that shown on FBD.

Ans.



Ans.



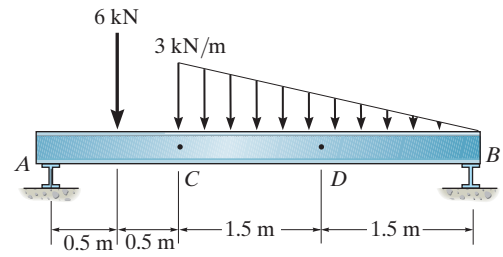
Ans.

Ans.

Ans:

$$P = 0.533 \text{ kN}, \quad N_C = -2.00 \text{ kN}, \quad V_C = -0.533 \text{ kN}, \\ M_C = 0.400 \text{ kN} \cdot \text{m}$$

*1-8. Determine the resultant internal loadings on the cross section through point C. Assume the reactions at the supports A and B are vertical.



Referring to the FBD of the entire beam, Fig. a,

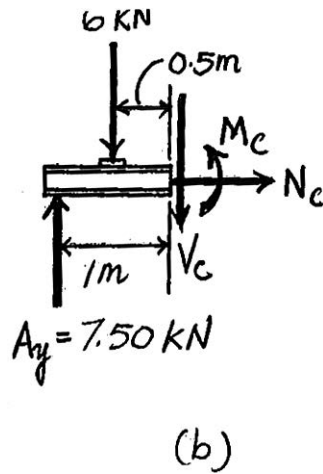
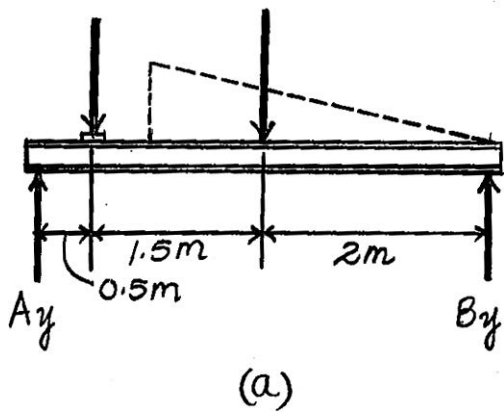
$$\zeta + \Sigma M_B = 0; \quad -A_y(4) + 6(3.5) + \frac{1}{2}(3)(3)(2) = 0 \quad A_y = 7.50 \text{ kN}$$

Referring to the FBD of this segment, Fig. b,

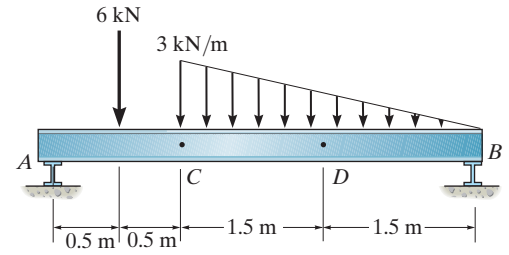
$$\rightarrow \Sigma F_x = 0; \quad N_C = 0 \quad \text{Ans.}$$

$$+\uparrow \Sigma F_y = 0; \quad 7.50 - 6 - V_C = 0 \quad V_C = 1.50 \text{ kN} \quad \text{Ans.}$$

$$\zeta + \Sigma M_C = 0; \quad M_C + 6(0.5) - 7.5(1) = 0 \quad M_C = 4.50 \text{ kN} \cdot \text{m} \quad \text{Ans.}$$



1-9. Determine the resultant internal loadings on the cross section through point D . Assume the reactions at the supports A and B are vertical.

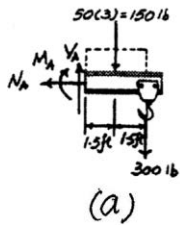


Referring to the FBD of the entire beam, Fig. a ,

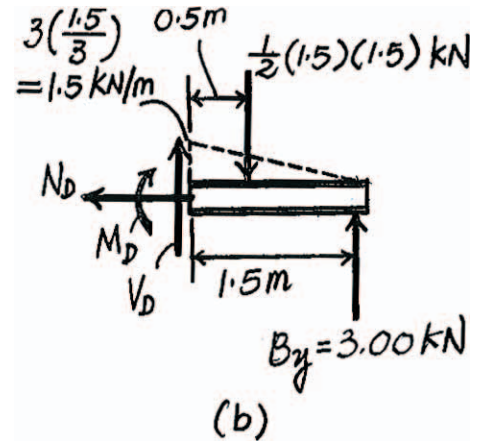
$$\zeta + \sum M_A = 0; \quad B_y(4) - 6(0.5) - \frac{1}{2}(3)(3)(2) = 0 \quad B_y = 3.00 \text{ kN}$$

Referring to the FBD of this segment, Fig. b ,

$$\rightarrow \sum F_x = 0; \quad N_D = 0$$



Ans.



$$+\uparrow \sum F_y = 0; \quad V_D - \frac{1}{2}(1.5)(1.5) + 3.00 = 0 \quad V_D = -1.875 \text{ kN} \quad \text{Ans.}$$

$$\zeta + \sum M_D = 0; \quad 3.00(1.5) - \frac{1}{2}(1.5)(1.5)(0.5) - M_D = 0 \quad M_D = 3.9375 \text{ kN} \cdot \text{m}$$

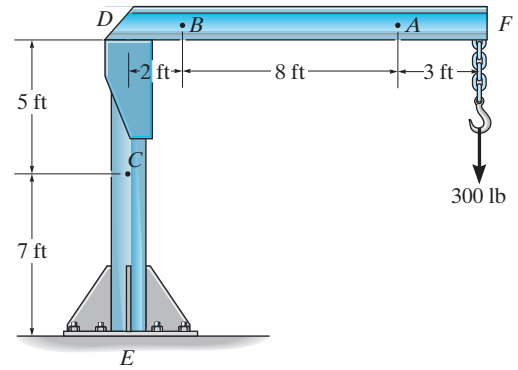
$$= 3.94 \text{ kN} \cdot \text{m} \quad \text{Ans.}$$

Ans:

$$N_D = 0, V_D = -1.875 \text{ kN},$$

$$M_D = 3.94 \text{ kN} \cdot \text{m}$$

1-10. The boom DF of the jib crane and the column DE have a uniform weight of 50 lb/ft. If the hoist and load weigh 300 lb, determine the resultant internal loadings in the crane on cross sections through points A , B , and C .



Equations of Equilibrium: For point A

$$\leftarrow \sum F_x = 0; \quad N_A = 0$$

$$+\uparrow \sum F_y = 0; \quad V_A - 150 - 300 = 0$$

$$V_A = 450 \text{ lb}$$

$$\zeta + \sum M_A = 0; \quad -M_A - 150(1.5) - 300(3) = 0$$

$$M_A = -1125 \text{ lb} \cdot \text{ft} = -1.125 \text{ kip} \cdot \text{ft}$$

Negative sign indicates that M_A acts in the opposite direction to that shown on FBD.

Equations of Equilibrium: For point B

$$\leftarrow \sum F_x = 0; \quad N_B = 0$$

$$+\uparrow \sum F_y = 0; \quad V_B - 550 - 300 = 0$$

$$V_B = 850 \text{ lb}$$

$$\zeta + \sum M_B = 0; \quad -M_B - 550(5.5) - 300(11) = 0$$

$$M_B = -6325 \text{ lb} \cdot \text{ft} = -6.325 \text{ kip} \cdot \text{ft}$$

Negative sign indicates that M_B acts in the opposite direction to that shown on FBD.

Equations of Equilibrium: For point C

$$\leftarrow \sum F_x = 0; \quad V_C = 0$$

$$+\uparrow \sum F_y = 0; \quad -N_C - 250 - 650 - 300 = 0$$

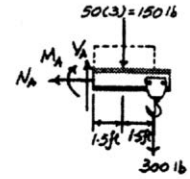
$$N_C = -1200 \text{ lb} = -1.20 \text{ kip}$$

$$\zeta + \sum M_C = 0; \quad -M_C - 650(6.5) - 300(13) = 0$$

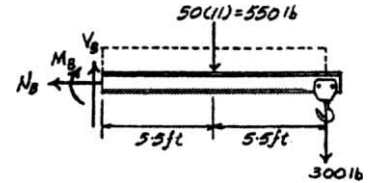
$$M_C = -8125 \text{ lb} \cdot \text{ft} = -8.125 \text{ kip} \cdot \text{ft}$$

Negative signs indicate that N_C and M_C act in the opposite direction to that shown on FBD.

Ans.

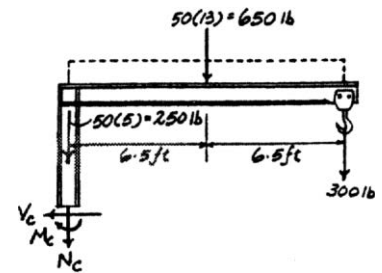


Ans.



Ans.

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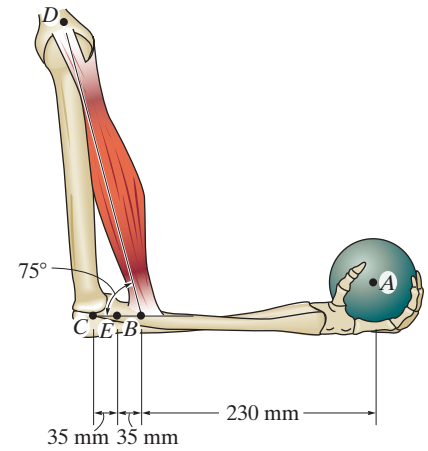
Ans:

$$N_A = 0, V_A = 450 \text{ lb}, M_A = -1.125 \text{ kip} \cdot \text{ft},$$

$$N_B = 0, V_B = 850 \text{ lb}, M_B = -6.325 \text{ kip} \cdot \text{ft},$$

$$V_C = 0, N_C = -1.20 \text{ kip}, M_C = -8.125 \text{ kip} \cdot \text{ft}$$

1-11. The forearm and biceps support the 2-kg load at *A*. If *C* can be assumed as a pin support, determine the resultant internal loadings acting on the cross section of the bone of the forearm at *E*. The biceps pulls on the bone along *BD*.



Support Reactions: In this case, all the support reactions will be completed. Referring to the free-body diagram of the forearm, Fig. *a*,

$$\zeta + \Sigma M_C = 0; \quad F_{BD} \sin 75^\circ(0.07) - 2(9.81)(0.3) = 0 \quad F_{BD} = 87.05 \text{ N}$$

$$\rightarrow \Sigma F_x = 0; \quad C_x - 87.05 \cos 75^\circ = 0 \quad C_x = 22.53 \text{ N}$$

$$+\uparrow \Sigma F_y = 0; \quad 87.05 \sin 75^\circ - 2(9.81) - C_y = 0 \quad C_y = 64.47 \text{ N}$$

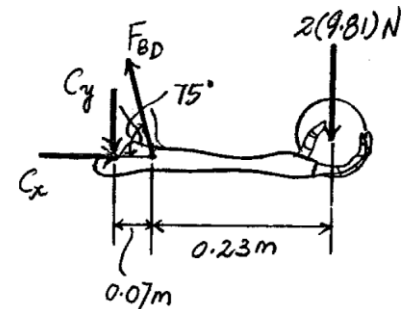
Internal Loadings: Using the results of C_x and C_y , section *CE* of the forearm will be considered. Referring to the free-body diagram of this part shown in Fig. *b*,

$$\rightarrow \Sigma F_x = 0; \quad N_E + 22.53 = 0 \quad N_E = -22.5 \text{ N} \quad \text{Ans.}$$

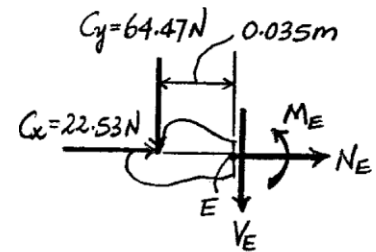
$$+\uparrow \Sigma F_y = 0; \quad -V_E - 64.47 = 0 \quad V_E = -64.5 \text{ N} \quad \text{Ans.}$$

$$\zeta + \Sigma M_E = 0; \quad M_E + 64.47(0.035) = 0 \quad M_E = -2.26 \text{ N} \cdot \text{m} \quad \text{Ans.}$$

The negative signs indicate that N_E , V_E and M_E act in the opposite sense to that shown on the free-body diagram.



(a)

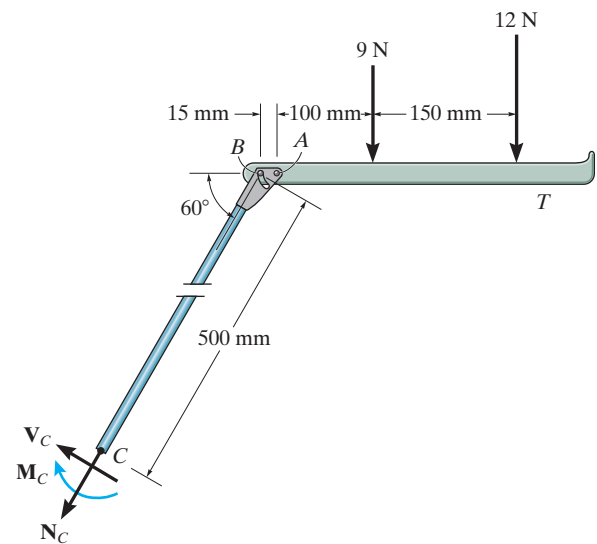


(b)

Ans:

$$N_E = -22.5 \text{ N}, V_E = -64.5 \text{ N}, M_E = -2.26 \text{ N} \cdot \text{m}$$

***1-12.** The serving tray T used on an airplane is supported on *each side* by an arm. The tray is pin connected to the arm at A , and at B there is a smooth pin. (The pin can move within the slot in the arms to permit folding the tray against the front passenger seat when not in use.) Determine the resultant internal loadings acting on the cross section of the arm through point C when the tray arm supports the loads shown.

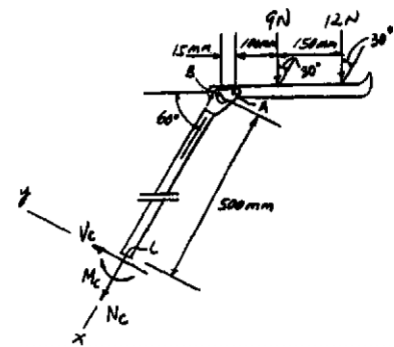


$$\rightarrow + \Sigma F_x = 0; \quad N_C + 9 \cos 30^\circ + 12 \cos 30^\circ = 0; \quad N_C = -18.2 \text{ N} \quad \text{Ans.}$$

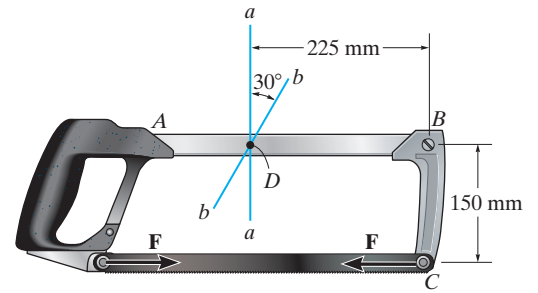
$$\uparrow + \Sigma F_y = 0; \quad V_C - 9 \sin 30^\circ - 12 \sin 30^\circ = 0; \quad V_C = 10.5 \text{ N} \quad \text{Ans.}$$

$$\curvearrow + \Sigma M_C = 0; \quad -M_C - 9(0.5 \cos 60^\circ + 0.115) - 12(0.5 \cos 60^\circ + 0.265) = 0$$

$$M_C = -9.46 \text{ N} \cdot \text{m} \quad \text{Ans.}$$



1-13. The blade of the hacksaw is subjected to a pretension force of $F = 100\text{ N}$. Determine the resultant internal loadings acting on section $a-a$ that passes through point D .

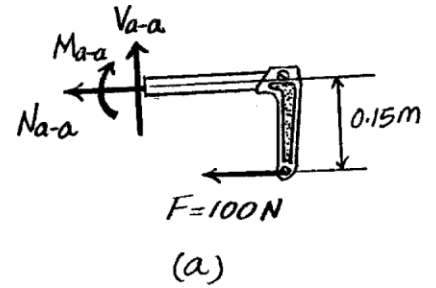


Internal Loadings: Referring to the free-body diagram of the section of the hacksaw shown in Fig. a ,

$$\leftarrow \Sigma F_x = 0; \quad N_{a-a} + 100 = 0 \quad N_{a-a} = -100\text{ N} \quad \text{Ans.}$$

$$+\uparrow \Sigma F_y = 0; \quad V_{a-a} = 0 \quad \text{Ans.}$$

$$\zeta + \Sigma M_D = 0; \quad -M_{a-a} - 100(0.15) = 0 \quad M_{a-a} = -15\text{ N} \cdot \text{m} \quad \text{Ans.}$$

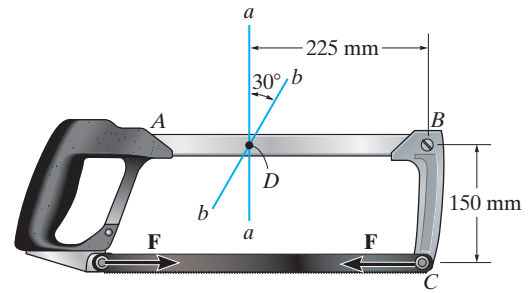


The negative sign indicates that N_{a-a} and M_{a-a} act in the opposite sense to that shown on the free-body diagram.

Ans:

$$N_{a-a} = -100\text{ N}, V_{a-a} = 0, M_{a-a} = -15\text{ N} \cdot \text{m}$$

1-14. The blade of the hacksaw is subjected to a pretension force of $F = 100\text{ N}$. Determine the resultant internal loadings acting on section $b-b$ that passes through point D .



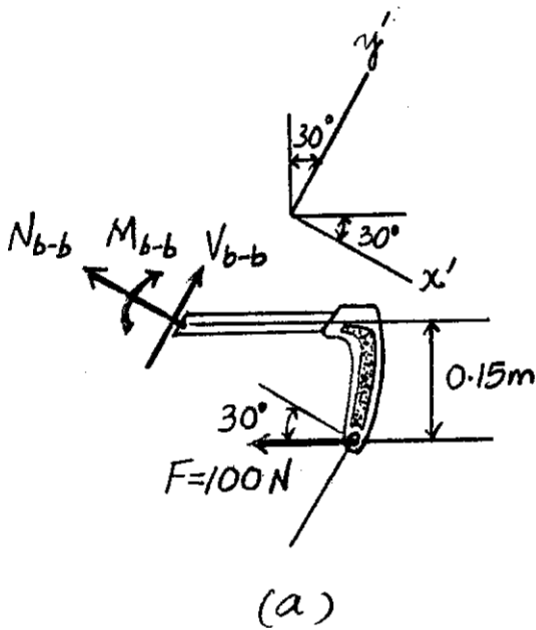
Internal Loadings: Referring to the free-body diagram of the section of the hacksaw shown in Fig. a ,

$$\Sigma F_{x'} = 0; \quad N_{b-b} + 100 \cos 30^\circ = 0 \quad N_{b-b} = -86.6\text{ N} \quad \text{Ans.}$$

$$\Sigma F_{y'} = 0; \quad V_{b-b} - 100 \sin 30^\circ = 0 \quad V_{b-b} = 50\text{ N} \quad \text{Ans.}$$

$$\zeta + \Sigma M_D = 0; \quad -M_{b-b} - 100(0.15) = 0 \quad M_{b-b} = -15\text{ N} \cdot \text{m} \quad \text{Ans.}$$

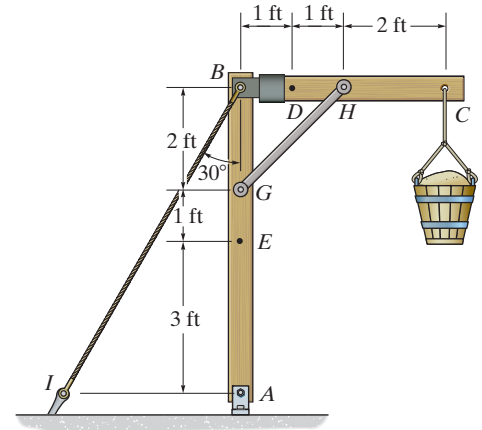
The negative sign indicates that N_{b-b} and M_{b-b} act in the opposite sense to that shown on the free-body diagram.



Ans:

$$N_{b-b} = -86.6\text{ N}, \quad V_{b-b} = 50\text{ N}, \quad M_{b-b} = -15\text{ N} \cdot \text{m}$$

1-15. A 150-lb bucket is suspended from a cable on the wooden frame. Determine the resultant internal loadings on the cross section at D .



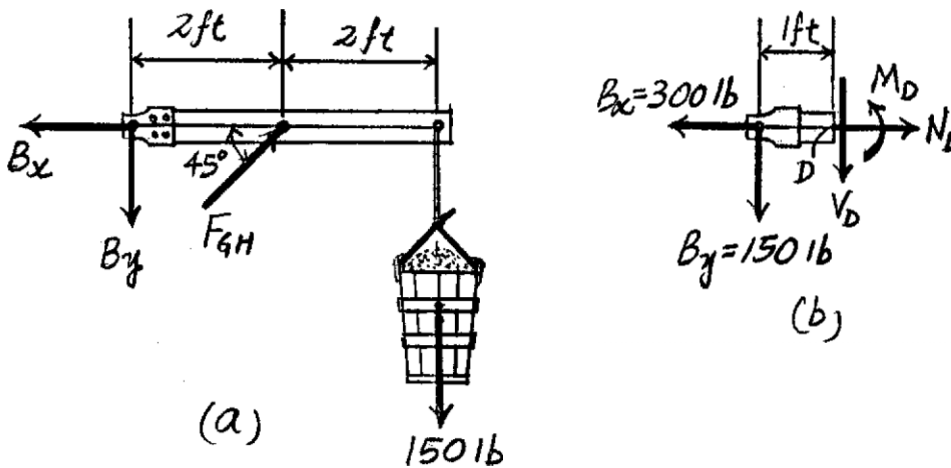
Support Reactions: We will only need to compute \mathbf{B}_x , \mathbf{B}_y , and \mathbf{F}_{GH} . Referring to the free-body diagram of member BC , Fig. a ,

$$\begin{aligned} \zeta + \Sigma M_B = 0: & \quad F_{GH} \sin 45^\circ(2) - 150(4) = 0 & \quad F_{GH} = 424.26 \text{ lb} \\ \rightarrow \Sigma F_x = 0; & \quad 424.26 \cos 45^\circ - B_x = 0 & \quad B_x = 300 \text{ lb} \\ + \uparrow \Sigma F_y = 0; & \quad 424.26 \sin 45^\circ - 150 - B_y = 0 & \quad B_y = 150 \text{ lb} \end{aligned}$$

Internal Loadings: Using the results of \mathbf{B}_x and \mathbf{B}_y , section BD of member BC will be considered. Referring to the free-body diagram of this part shown in Fig. b ,

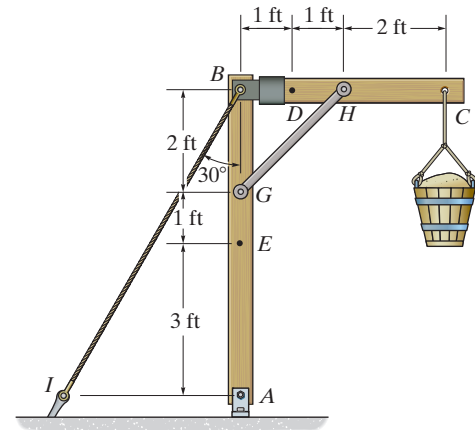
$$\begin{aligned} \rightarrow \Sigma F_x = 0; & \quad N_D - 300 = 0 & \quad N_D = 300 \text{ lb} & \quad \text{Ans.} \\ + \uparrow \Sigma F_y = 0; & \quad -V_D - 150 = 0 & \quad V_D = -150 \text{ lb} & \quad \text{Ans.} \\ \zeta + \Sigma M_D = 0; & \quad 150(1) + M_D = 0 & \quad M_D = -150 \text{ lb} \cdot \text{ft} & \quad \text{Ans.} \end{aligned}$$

The negative signs indicates that \mathbf{V}_D and \mathbf{M}_D act in the opposite sense to that shown on the free-body diagram.



Ans:
 $N_D = 300 \text{ lb}, V_D = -150 \text{ lb}, M_D = -150 \text{ lb} \cdot \text{ft}$

*1-16. A 150-lb bucket is suspended from a cable on the wooden frame. Determine the resultant internal loadings acting on the cross section at E .



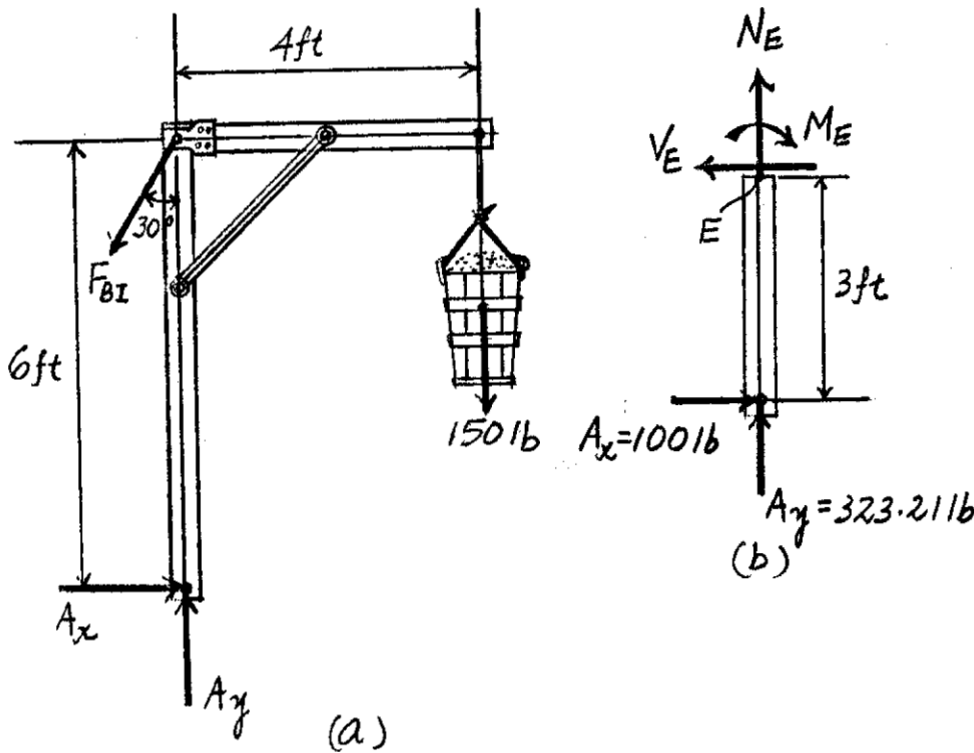
Support Reactions: We will only need to compute A_x , A_y , and F_{BI} . Referring to the free-body diagram of the frame, Fig. a ,

$$\begin{aligned} \zeta + \Sigma M_A = 0; & \quad F_{BI} \sin 30^\circ(6) - 150(4) = 0 & \quad F_{BI} = 200 \text{ lb} \\ \rightarrow \Sigma F_x = 0; & \quad A_x - 200 \sin 30^\circ = 0 & \quad A_x = 100 \text{ lb} \\ + \uparrow \Sigma F_y = 0; & \quad A_y - 200 \cos 30^\circ - 150 = 0 & \quad A_y = 323.21 \text{ lb} \end{aligned}$$

Internal Loadings: Using the results of A_x and A_y , section AE of member AB will be considered. Referring to the free-body diagram of this part shown in Fig. b ,

$$\begin{aligned} \rightarrow \Sigma F_x = 0; & \quad N_E + 323.21 = 0 & \quad N_E = -323 \text{ lb} & \quad \text{Ans.} \\ + \uparrow \Sigma F_y = 0; & \quad 100 - V_E = 0 & \quad V_E = 100 \text{ lb} & \quad \text{Ans.} \\ \zeta + \Sigma M_D = 0; & \quad 100(3) - M_E = 0 & \quad M_E = 300 \text{ lb} \cdot \text{ft} & \quad \text{Ans.} \end{aligned}$$

The negative sign indicates that N_E acts in the opposite sense to that shown on the free-body diagram.



1-17. Determine resultant internal loadings acting on section *a-a* and section *b-b*. Each section passes through the centerline at point *C*.

Referring to the FBD of the entire beam, Fig. *a*,

$$\zeta + \Sigma M_A = 0; \quad N_B \sin 45^\circ(6) - 5(4.5) = 0 \quad N_B = 5.303 \text{ kN}$$

Referring to the FBD of this segment (section *a-a*), Fig. *b*,

$$+\nearrow \Sigma F_x = 0; \quad N_{a-a} + 5.303 \cos 45^\circ = 0 \quad N_{a-a} = -3.75 \text{ kN}$$

$$+\searrow \Sigma F_y = 0; \quad V_{a-a} + 5.303 \sin 45^\circ - 5 = 0 \quad V_{a-a} = 1.25 \text{ kN}$$

$$\zeta + \Sigma M_C = 0; \quad 5.303 \sin 45^\circ(3) - 5(1.5) - M_{a-a} = 0 \quad M_{a-a} = 3.75 \text{ kN} \cdot \text{m}$$

Referring to the FBD (section *b-b*) in Fig. *c*,

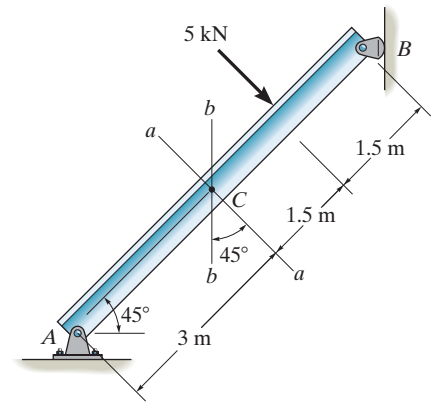
$$\leftarrow \Sigma F_x = 0; \quad N_{b-b} - 5 \cos 45^\circ + 5.303 = 0 \quad N_{b-b} = -1.768 \text{ kN}$$

$$= -1.77 \text{ kN}$$

$$+\uparrow \Sigma F_y = 0; \quad V_{b-b} - 5 \sin 45^\circ = 0 \quad V_{b-b} = 3.536 \text{ kN} = 3.54 \text{ kN}$$

$$\zeta + \Sigma M_C = 0; \quad 5.303 \sin 45^\circ(3) - 5(1.5) - M_{b-b} = 0$$

$$M_{b-b} = 3.75 \text{ kN} \cdot \text{m}$$



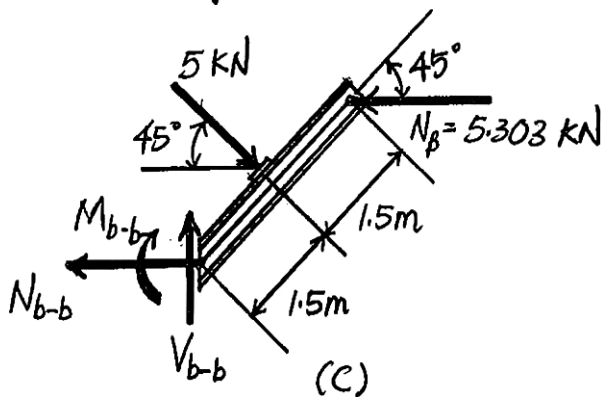
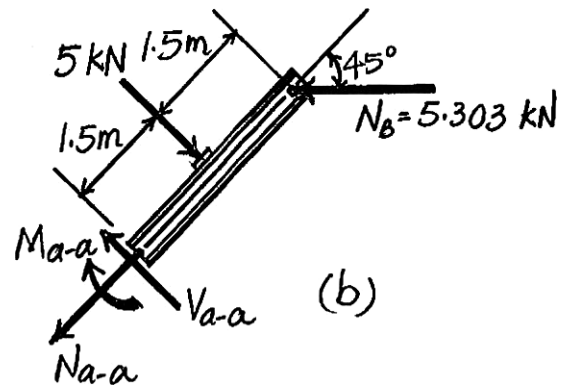
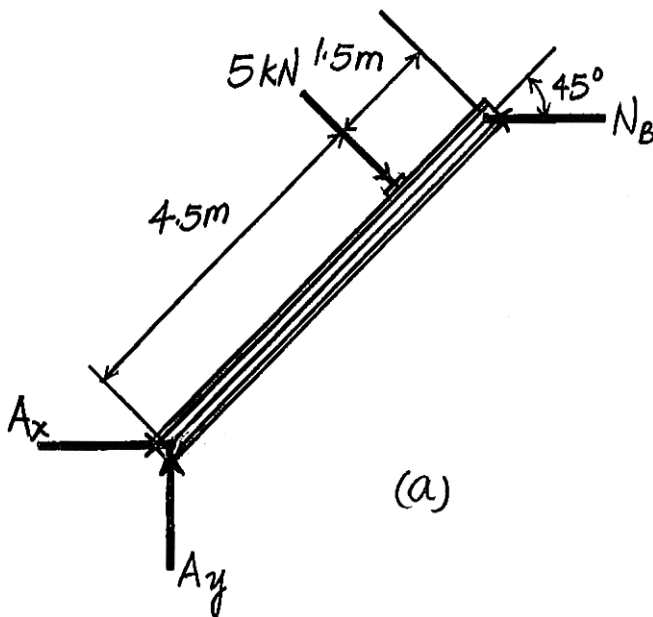
Ans.

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Ans.



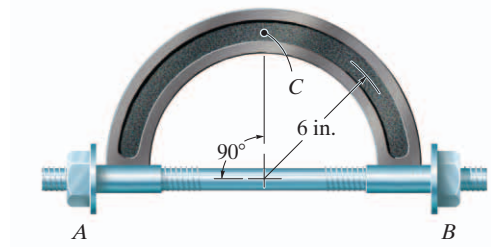
Ans:

$$N_{a-a} = -3.75 \text{ kN}, \quad V_{a-a} = 1.25 \text{ kN},$$

$$M_{a-a} = 3.75 \text{ kN} \cdot \text{m}, \quad N_{b-b} = -1.77 \text{ kN},$$

$$V_{b-b} = 3.54 \text{ kN} \cdot \text{m}, \quad M_{b-b} = 3.75 \text{ kN} \cdot \text{m}$$

1-18. The bolt shank is subjected to a tension of 80 lb. Determine the resultant internal loadings acting on the cross section at point C.



Segment AC:

$$\rightarrow \Sigma F_x = 0; \quad N_C + 80 = 0; \quad N_C = -80 \text{ lb}$$

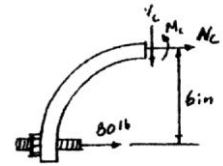
$$+\uparrow \Sigma F_y = 0; \quad V_C = 0$$

$$\zeta + \Sigma M_C = 0; \quad M_C + 80(6) = 0; \quad M_C = -480 \text{ lb} \cdot \text{in.}$$

Ans.

Ans.

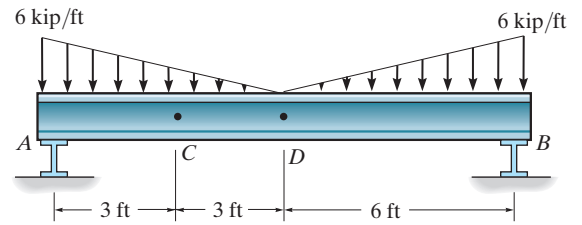
Ans.



Ans:

$$N_C = -80 \text{ lb}, V_C = 0, M_C = -480 \text{ lb} \cdot \text{in.}$$

1-19. Determine the resultant internal loadings acting on the cross section through point *C*. Assume the reactions at the supports *A* and *B* are vertical.



Referring to the FBD of the entire beam, Fig. *a*,

$$\zeta + \Sigma M_B = 0; \quad \frac{1}{2}(6)(6)(2) + \frac{1}{2}(6)(6)(10) - A_y(12) = 0 \quad A_y = 18.0 \text{ kip}$$

Referring to the FBD of this segment, Fig. *b*,

$$\rightarrow \Sigma F_x = 0; \quad N_C = 0$$

Ans.

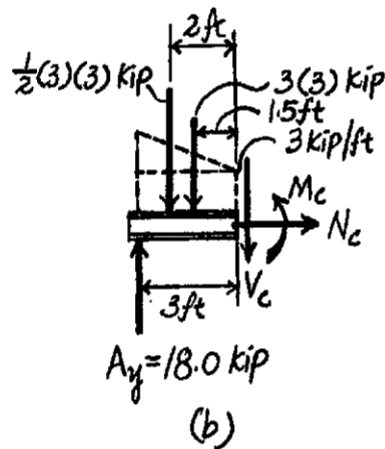
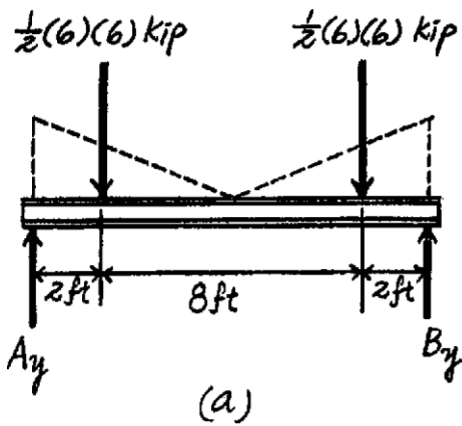
$$+\uparrow \Sigma F_y = 0; \quad 18.0 - \frac{1}{2}(3)(3) - (3)(3) - V_C = 0 \quad V_C = 4.50 \text{ kip}$$

Ans.

$$\zeta + \Sigma M_C = 0; \quad M_C + (3)(3)(1.5) + \frac{1}{2}(3)(3)(2) - 18.0(3) = 0$$

$$M_C = 31.5 \text{ kip} \cdot \text{ft}$$

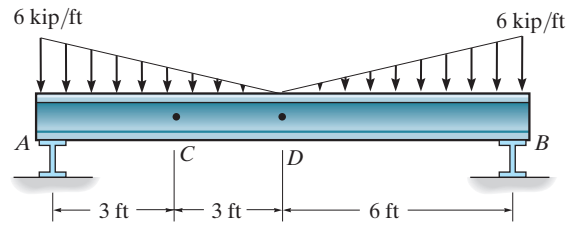
Ans.



Ans:

$$N_C = 0, V_C = 4.50 \text{ kip}, M_C = 31.5 \text{ kip} \cdot \text{ft}$$

*1-20. Determine the resultant internal loadings acting on the cross section through point D . Assume the reactions at the supports A and B are vertical.



Referring to the FBD of the entire beam, Fig. a ,

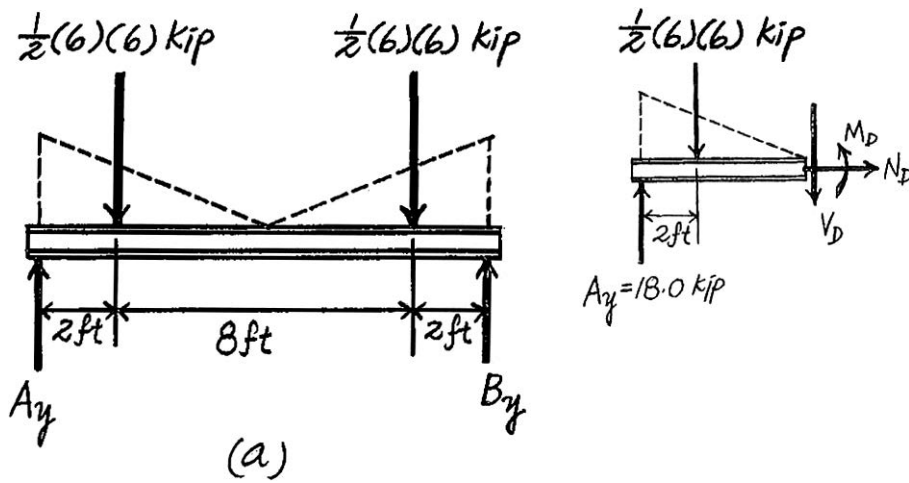
$$\zeta + \Sigma M_B = 0; \quad \frac{1}{2}(6)(6)(2) + \frac{1}{2}(6)(6)(10) - A_y(12) = 0 \quad A_y = 18.0 \text{ kip}$$

Referring to the FBD of this segment, Fig. b ,

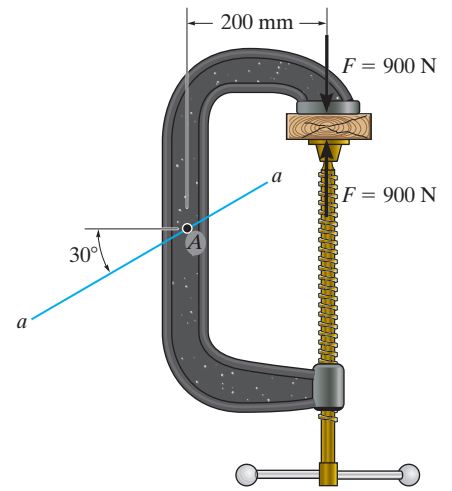
$$\rightarrow \Sigma F_x = 0; \quad N_D = 0 \quad \text{Ans.}$$

$$+\uparrow \Sigma F_y = 0; \quad 18.0 - \frac{1}{2}(6)(6) - V_D = 0 \quad V_D = 0 \quad \text{Ans.}$$

$$\zeta + \Sigma M_A = 0; \quad M_D - 18.0(2) = 0 \quad M_D = 36.0 \text{ kip} \cdot \text{ft} \quad \text{Ans.}$$



1-21. The forged steel clamp exerts a force of $F = 900\text{ N}$ on the wooden block. Determine the resultant internal loadings acting on section $a-a$ passing through point A .

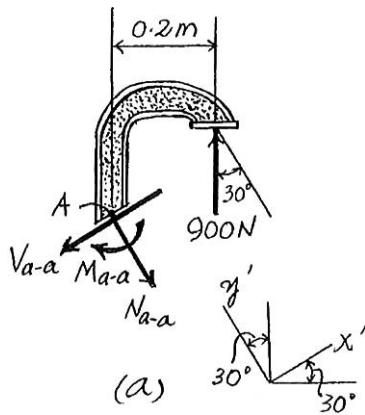


Internal Loadings: Referring to the free-body diagram of the section of the clamp shown in Fig. a ,

$$\Sigma F_{y'} = 0; \quad 900 \cos 30^\circ - N_{a-a} = 0 \quad N_{a-a} = 779\text{ N} \quad \text{Ans.}$$

$$\Sigma F_{x'} = 0; \quad V_{a-a} - 900 \sin 30^\circ = 0 \quad V_{a-a} = 450\text{ N} \quad \text{Ans.}$$

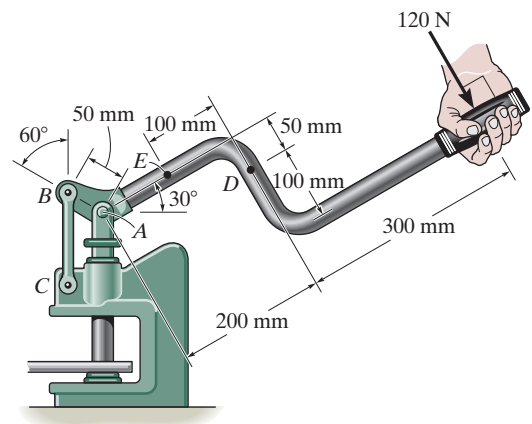
$$\zeta + \Sigma M_A = 0; \quad 900(0.2) - M_{a-a} = 0 \quad M_{a-a} = 180\text{ N}\cdot\text{m} \quad \text{Ans.}$$



Ans:

$$N_{a-a} = 779\text{ N}, V_{a-a} = 450\text{ N}, M_{a-a} = 180\text{ N}\cdot\text{m}:$$

1-22. The metal stud punch is subjected to a force of 120 N on the handle. Determine the magnitude of the reactive force at the pin A and in the short link BC . Also, determine the internal resultant loadings acting on the cross section passing through the handle arm at D .



Member:

$$\zeta + \sum M_A = 0; \quad F_{BC} \cos 30^\circ (50) - 120(500) = 0$$

$$F_{BC} = 1385.6 \text{ N} = 1.39 \text{ kN}$$

$$+\uparrow \sum F_y = 0; \quad A_y - 1385.6 - 120 \cos 30^\circ = 0$$

$$A_y = 1489.56 \text{ N}$$

$$\leftarrow \sum F_x = 0; \quad A_x - 120 \sin 30^\circ = 0; \quad A_x = 60 \text{ N}$$

$$F_A = \sqrt{1489.56^2 + 60^2}$$

$$= 1491 \text{ N} = 1.49 \text{ kN}$$

Segment:

$$\nearrow \sum F_{x'} = 0; \quad N_D - 120 = 0$$

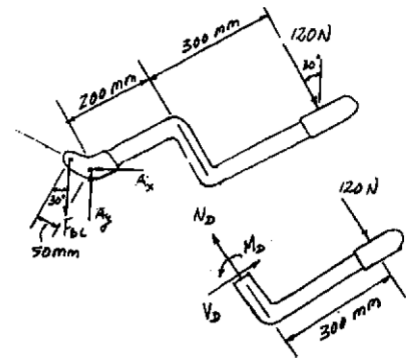
$$N_D = 120 \text{ N}$$

$$+\nearrow \sum F_{y'} = 0; \quad V_D = 0$$

$$\zeta + \sum M_D = 0; \quad M_D - 120(0.3) = 0$$

$$M_D = 36.0 \text{ N} \cdot \text{m}$$

Ans.



Ans.

Ans.

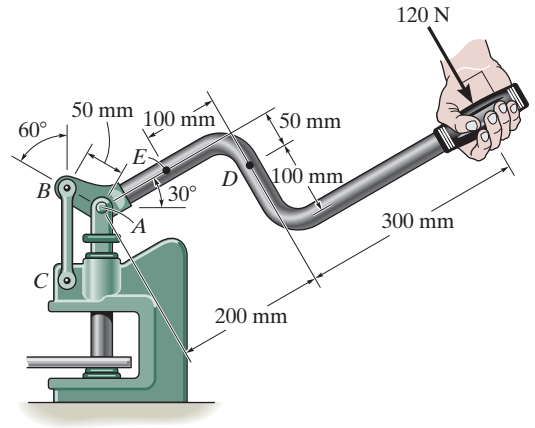
Ans.

Ans.

Ans:

$$F_{BC} = 1.39 \text{ kN}, F_A = 1.49 \text{ kN}, N_D = 120 \text{ N}, \\ V_D = 0, M_D = 36.0 \text{ N} \cdot \text{m}$$

1–23. Solve Prob. 1–22 for the resultant internal loadings acting on the cross section passing through the handle arm at E and at a cross section of the short link BC .



Member:

$$\zeta + \Sigma M_A = 0; \quad F_{BC} \cos 30^\circ(50) - 120(500) = 0$$

$$F_{BC} = 1385.6 \text{ N} = 1.3856 \text{ kN}$$

Segment:

$$\uparrow \Sigma F_{x'} = 0; \quad N_E = 0$$

$$\curvearrowleft + \Sigma F_{y'} = 0; \quad V_E - 120 = 0; \quad V_E = 120 \text{ N}$$

$$\zeta + \Sigma M_E = 0; \quad M_E - 120(0.4) = 0; \quad M_E = 48.0 \text{ N} \cdot \text{m}$$

Short link:

$$\leftarrow \Sigma F_x = 0; \quad V = 0$$

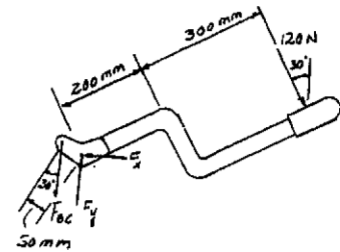
$$+\uparrow \Sigma F_y = 0; \quad 1.3856 - N = 0; \quad N = 1.39 \text{ kN}$$

$$\zeta + \Sigma M_H = 0; \quad M = 0$$

Ans.

Ans.

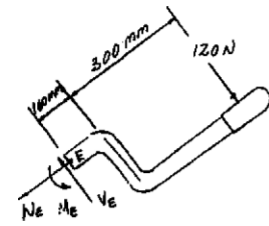
Ans.



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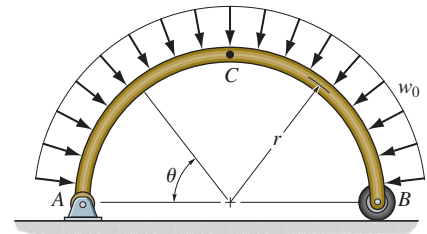


Ans:

$$N_E = 0, V_E = 120 \text{ N}, M_E = 48.0 \text{ N} \cdot \text{m},$$

$$\text{Short link: } V = 0, N = 1.39 \text{ kN}, M = 0$$

*1-24. Determine the resultant internal loadings acting on the cross section of the semicircular arch at C.



$$\zeta + \Sigma M_A = 0; \quad B_y(2r) - \int_0^\pi (w_0 r d\theta)(\cos \theta)(r \sin \theta) - \int_0^\pi (w_0 r d\theta)(\sin \theta)r(1 - \cos \theta) = 0$$

$$B_y(2r) - w_0 r^2 \int_0^\pi \sin \theta d\theta = 0$$

$$B_y(2r) - w_0 r^2(-\cos \theta)\Big|_0^\pi = 0$$

$$B_y = w_0 r$$

$$\rightarrow \Sigma F_x = 0; \quad -N_C - w_0 r \int_0^{\pi/2} \cos \theta d\theta = 0$$

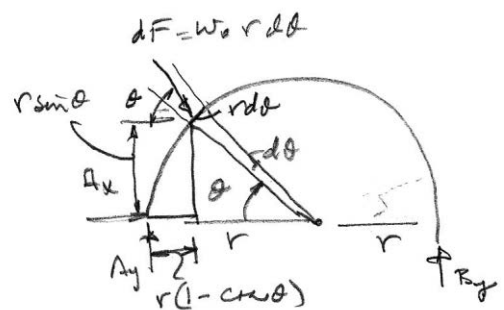
$$N_C = -w_0 r \sin \theta \Big|_0^{\pi/2} = -w_0 r$$

$$+\uparrow \Sigma F_y = 0; \quad w_0 r + V_C - w_0 r \int_0^{\pi/2} \sin \theta d\theta = 0$$

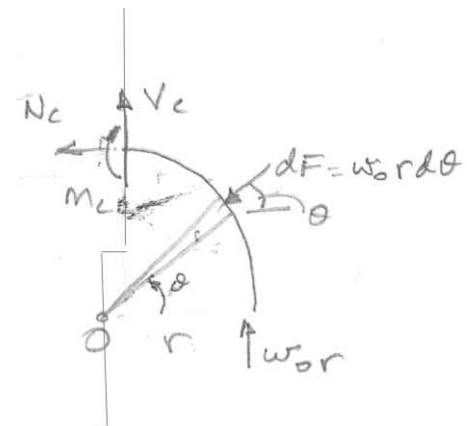
$$w_0 r + V_C - w_0 r(-\cos \theta) \Big|_0^{\pi/2} = 0; \quad V_2 = 0$$

$$\zeta + \Sigma M_0 = 0; \quad w_0 r(r) - M_C + (-w_0 r)(r) = 0$$

$$M_C = 0$$



Ans.



Ans.

Ans.

1-25. Determine the resultant internal loadings acting on the cross section through point *B* of the signpost. The post is fixed to the ground and a uniform pressure of 7 lb/ft² acts perpendicular to the face of the sign.

$$\Sigma F_x = 0; \quad (V_B)_x - 105 = 0; \quad (V_B)_x = 105 \text{ lb}$$

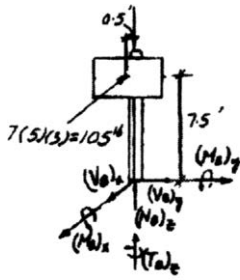
$$\Sigma F_y = 0; \quad (V_B)_y = 0$$

$$\Sigma F_z = 0; \quad (N_B)_z = 0$$

$$\Sigma M_x = 0; \quad (M_B)_x = 0$$

$$\Sigma M_y = 0; \quad (M_B)_y - 105(7.5) = 0; \quad (M_B)_y = 788 \text{ lb} \cdot \text{ft}$$

$$\Sigma M_z = 0; \quad (T_B)_z - 105(0.5) = 0; \quad (T_B)_z = 52.5 \text{ lb} \cdot \text{ft}$$



Ans.

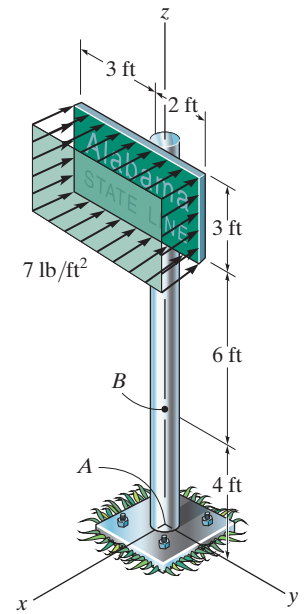
Ans.

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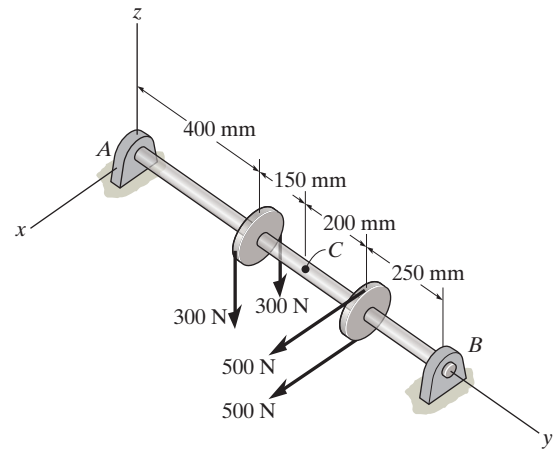
Ans:

$$(V_B)_x = 105 \text{ lb}, \quad (V_B)_y = 0, \quad (N_B)_z = 0,$$

$$(M_B)_x = 0, \quad (M_B)_y = 788 \text{ lb} \cdot \text{ft},$$

$$(T_B)_z = 52.5 \text{ lb} \cdot \text{ft}$$

1-26. The shaft is supported at its ends by two bearings *A* and *B* and is subjected to the forces applied to the pulleys fixed to the shaft. Determine the resultant internal loadings acting on the cross section located at point *C*. The 300-N forces act in the $-z$ direction and the 500-N forces act in the $+x$ direction. The journal bearings at *A* and *B* exert only x and z components of force on the shaft.



$$\Sigma F_x = 0; \quad (V_C)_x + 1000 - 750 = 0; \quad (V_C)_x = -250 \text{ N}$$

Ans.

$$\Sigma F_y = 0; \quad (N_C)_y = 0$$

Ans.

$$\Sigma F_z = 0; \quad (V_C)_z + 240 = 0; \quad (V_C)_z = -240 \text{ N}$$

Ans.

$$\Sigma M_x = 0; \quad (M_C)_x + 240(0.45) = 0; \quad (M_C)_x = -108 \text{ N}\cdot\text{m}$$

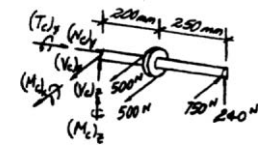
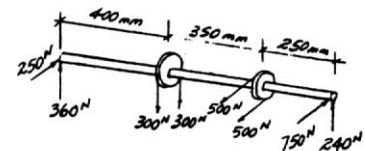
Ans.

$$\Sigma M_y = 0; \quad (T_C)_y = 0$$

Ans.

$$\Sigma M_z = 0; \quad (M_C)_z - 1000(0.2) + 750(0.45) = 0; \quad (M_C)_z = -138 \text{ N}\cdot\text{m}$$

Ans.



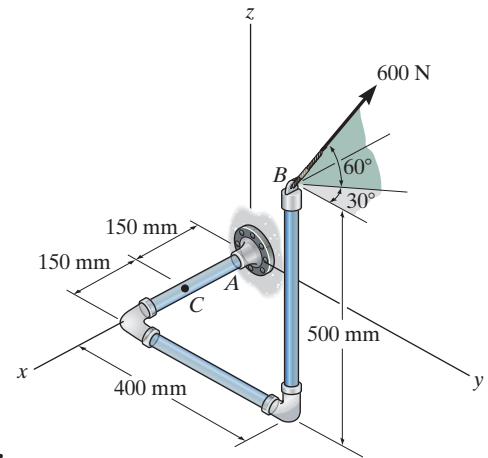
Ans:

$$(V_C)_x = -250 \text{ N}, \quad (N_C)_y = 0, \quad (V_C)_z = -240 \text{ N},$$

$$(M_C)_x = -108 \text{ N}\cdot\text{m}, \quad (T_C)_y = 0,$$

$$(M_C)_z = -138 \text{ N}\cdot\text{m}$$

1-27. The pipe assembly is subjected to a force of 600 N at *B*. Determine the resultant internal loadings acting on the cross section at *C*.



Internal Loading: Referring to the free-body diagram of the section of the pipe shown in Fig. *a*,

$$\Sigma F_x = 0; (N_C)_x - 600 \cos 60^\circ \sin 30^\circ = 0 \quad (N_C)_x = 150 \text{ N} \quad \text{Ans.}$$

$$\Sigma F_y = 0; (V_C)_y + 600 \cos 60^\circ \cos 30^\circ = 0 \quad (V_C)_y = -260 \text{ N} \quad \text{Ans.}$$

$$\Sigma F_z = 0; (V_C)_z + 600 \sin 60^\circ = 0 \quad (V_C)_z = -520 \text{ N} \quad \text{Ans.}$$

$$\Sigma M_x = 0; (T_C)_x + 600 \sin 60^\circ(0.4) - 600 \cos 60^\circ \cos 30^\circ(0.5) = 0$$

$$(T_C)_x = -77.9 \text{ N} \cdot \text{m} \quad \text{Ans.}$$

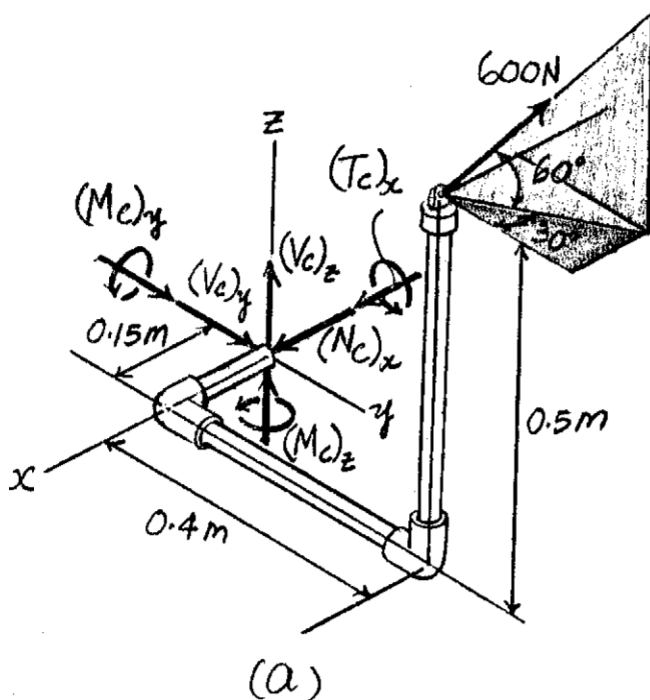
$$\Sigma M_y = 0; (M_C)_y - 600 \sin 60^\circ (0.15) - 600 \cos 60^\circ \sin 30^\circ(0.5) = 0$$

$$(M_C)_y = 153 \text{ N} \cdot \text{m} \quad \text{Ans.}$$

$$\Sigma M_z = 0; (M_C)_z + 600 \cos 60^\circ \cos 30^\circ(0.15) + 600 \cos 60^\circ \sin 30^\circ(0.4) = 0$$

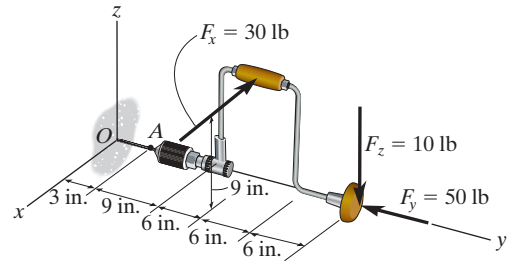
$$(M_C)_z = -99.0 \text{ N} \cdot \text{m} \quad \text{Ans.}$$

The negative signs indicate that $(V_C)_y$, $(V_C)_z$, $(T_C)_x$, and $(M_C)_z$ act in the opposite sense to that shown on the free-body diagram.



Ans:
 $(N_C)_x = 150 \text{ N}$, $(V_C)_y = -260 \text{ N}$,
 $(V_C)_z = -520 \text{ N}$, $(T_C)_x = -77.9 \text{ N} \cdot \text{m}$,
 $(M_C)_y = 153 \text{ N} \cdot \text{m}$, $(M_C)_z = -99.0 \text{ N} \cdot \text{m}$

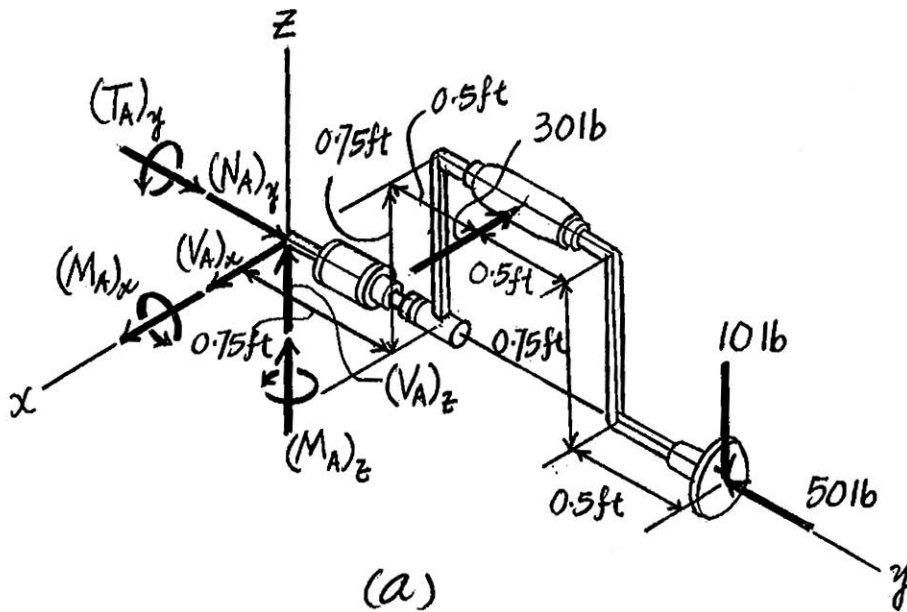
*1-28. The brace and drill bit is used to drill a hole at O . If the drill bit jams when the brace is subjected to the forces shown, determine the resultant internal loadings acting on the cross section of the drill bit at A .



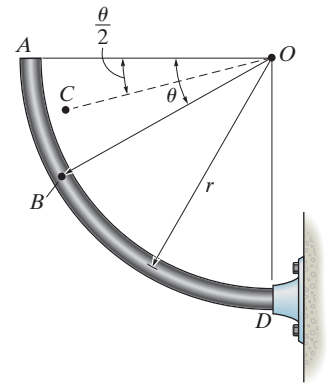
Internal Loading: Referring to the free-body diagram of the section of the drill and brace shown in Fig. a ,

$\Sigma F_x = 0;$	$(V_A)_x - 30 = 0$	$(V_A)_x = 30 \text{ lb}$	Ans.
$\Sigma F_y = 0;$	$(N_A)_y - 50 = 0$	$(N_A)_y = 50 \text{ lb}$	Ans.
$\Sigma F_z = 0;$	$(V_A)_z - 10 = 0$	$(V_A)_z = 10 \text{ lb}$	Ans.
$\Sigma M_x = 0;$	$(M_A)_x - 10(2.25) = 0$	$(M_A)_x = 22.5 \text{ lb} \cdot \text{ft}$	Ans.
$\Sigma M_y = 0;$	$(T_A)_y - 30(0.75) = 0$	$(T_A)_y = 22.5 \text{ lb} \cdot \text{ft}$	Ans.
$\Sigma M_z = 0;$	$(M_A)_z + 30(1.25) = 0$	$(M_A)_z = -37.5 \text{ lb} \cdot \text{ft}$	Ans.

The negative sign indicates that $(M_A)_z$ acts in the opposite sense to that shown on the free-body diagram.



1-29. The curved rod AD of radius r has a weight per length of w . If it lies in the vertical plane, determine the resultant internal loadings acting on the cross section through point B . *Hint:* The distance from the centroid C of segment AB to point O is $OC = [2r \sin(\theta/2)]/\theta$.



$$\checkmark \Sigma F_x = 0; \quad N_B + wr\theta \cos \theta = 0$$

$$N_B = -wr\theta \cos \theta$$

Ans.

$$+\nearrow \Sigma F_y = 0; \quad -V_B - wr\theta \sin \theta = 0$$

$$V_B = -wr\theta \sin \theta$$

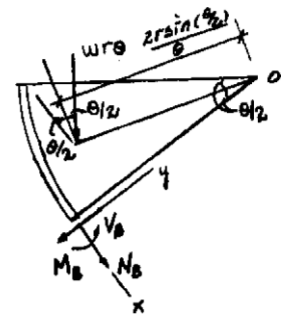
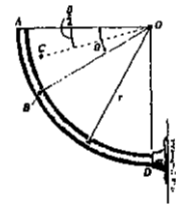
Ans.

$$\zeta + \Sigma M_O = 0; \quad wr\theta \left(\cos \frac{\theta}{2} \right) \left(\frac{2r \sin(\theta/2)}{\theta} \right) + (N_B)r + M_B = 0$$

$$M_B = -N_B r - wr^2 2 \sin(\theta/2) \cos(\theta/2)$$

$$M_B = wr^2(\theta \cos \theta - \sin \theta)$$

Ans.

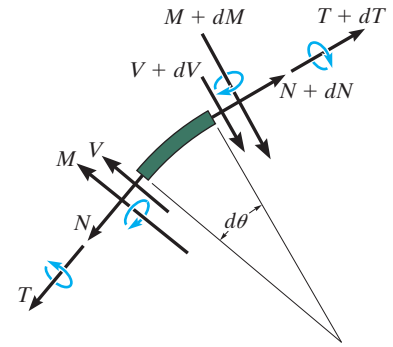


Ans:

$$N_B = -wr\theta \cos \theta, \quad V_B = -wr\theta \sin \theta,$$

$$M_B = wr^2(\theta \cos \theta - \sin \theta)$$

1-30. A differential element taken from a curved bar is shown in the figure. Show that $dN/d\theta = V$, $dV/d\theta = -N$, $dM/d\theta = -T$, and $dT/d\theta = M$.



$$\Sigma F_x = 0;$$

$$N \cos \frac{d\theta}{2} + V \sin \frac{d\theta}{2} - (N + dN) \cos \frac{d\theta}{2} + (V + dV) \sin \frac{d\theta}{2} = 0 \quad (1)$$

$$\Sigma F_y = 0;$$

$$N \sin \frac{d\theta}{2} - V \cos \frac{d\theta}{2} + (N + dN) \sin \frac{d\theta}{2} + (V + dV) \cos \frac{d\theta}{2} = 0 \quad (2)$$

$$\Sigma M_x = 0;$$

$$T \cos \frac{d\theta}{2} + M \sin \frac{d\theta}{2} - (T + dT) \cos \frac{d\theta}{2} + (M + dM) \sin \frac{d\theta}{2} = 0 \quad (3)$$

$$\Sigma M_y = 0;$$

$$T \sin \frac{d\theta}{2} - M \cos \frac{d\theta}{2} + (T + dT) \sin \frac{d\theta}{2} + (M + dM) \cos \frac{d\theta}{2} = 0 \quad (4)$$

Since $\frac{d\theta}{2}$ is small, then $\sin \frac{d\theta}{2} = \frac{d\theta}{2}$, $\cos \frac{d\theta}{2} = 1$

$$\text{Eq. (1) becomes } Vd\theta - dN + \frac{dVd\theta}{2} = 0$$

Neglecting the second order term, $Vd\theta - dN = 0$

$$\frac{dN}{d\theta} = V \quad \text{QED}$$

$$\text{Eq. (2) becomes } Nd\theta + dV + \frac{dNd\theta}{2} = 0$$

Neglecting the second order term, $Nd\theta + dV = 0$

$$\frac{dV}{d\theta} = -N \quad \text{QED}$$

$$\text{Eq. (3) becomes } Md\theta - dT + \frac{dMd\theta}{2} = 0$$

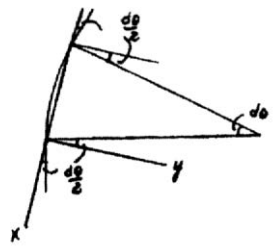
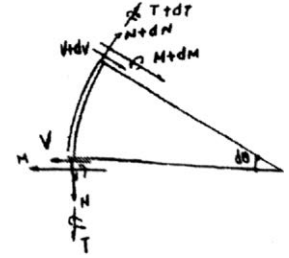
Neglecting the second order term, $Md\theta - dT = 0$

$$\frac{dT}{d\theta} = M \quad \text{QED}$$

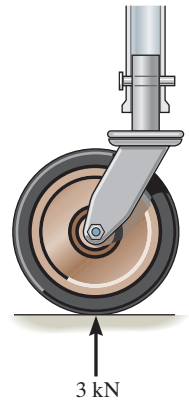
$$\text{Eq. (4) becomes } Td\theta + dM + \frac{dTd\theta}{2} = 0$$

Neglecting the second order term, $Td\theta + dM = 0$

$$\frac{dM}{d\theta} = -T \quad \text{QED}$$



1-31. The supporting wheel on a scaffold is held in place on the leg using a 4-mm-diameter pin as shown. If the wheel is subjected to a normal force of 3 kN, determine the average shear stress developed in the pin. Neglect friction between the inner scaffold puller leg and the tube used on the wheel.



$$+\uparrow \Sigma F_y = 0; \quad 3 \text{ kN} \cdot 2V = 0; \quad V = 1.5 \text{ kN}$$

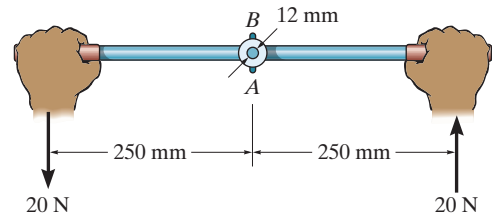
$$\tau_{\text{avg}} = \frac{V}{A} = \frac{1.5(10^3)}{\frac{\pi}{4}(0.004)^2} = 119 \text{ MPa}$$

Ans.



Ans:
 $\tau_{\text{avg}} = 119 \text{ MPa}$

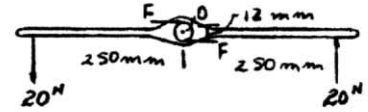
***1-32.** The lever is held to the fixed shaft using a tapered pin AB , which has a mean diameter of 6 mm. If a couple is applied to the lever, determine the average shear stress in the pin between the pin and lever.



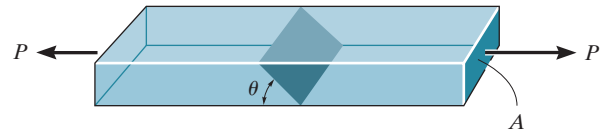
$$\zeta + \sum M_O = 0; \quad -F(12) + 20(500) = 0; \quad F = 833.33 \text{ N}$$

$$\tau_{\text{avg}} = \frac{V}{A} = \frac{833.33}{\frac{\pi(6)^2}{4(1000)}} = 29.5 \text{ MPa}$$

Ans.



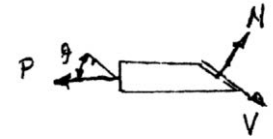
1-33. The bar has a cross-sectional area A and is subjected to the axial load P . Determine the average normal and average shear stresses acting over the shaded section, which is oriented at θ from the horizontal. Plot the variation of these stresses as a function of θ ($0 \leq \theta \leq 90^\circ$).



Equations of Equilibrium:

$$\downarrow + \Sigma F_x = 0; \quad V - P \cos \theta = 0 \quad V = P \cos \theta$$

$$\nearrow + \Sigma F_y = 0; \quad N - P \sin \theta = 0 \quad N = P \sin \theta$$

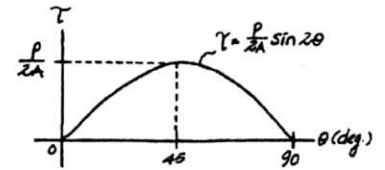


Average Normal Stress and Shear Stress: Area at θ plane, $A' = \frac{A}{\sin \theta}$.

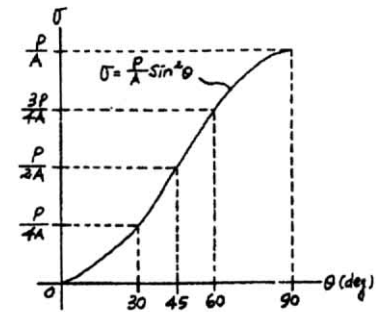
$$\sigma_{\text{avg}} = \frac{N}{A'} = \frac{P \sin \theta}{\frac{A}{\sin \theta}} = \frac{P}{A} \sin^2 \theta$$

$$\begin{aligned} \tau_{\text{avg}} &= \frac{V}{A'} = \frac{P \cos \theta}{\frac{A}{\sin \theta}} \\ &= \frac{P}{A} \sin \theta \cos \theta = \frac{P}{2A} \sin 2\theta \end{aligned}$$

Ans.



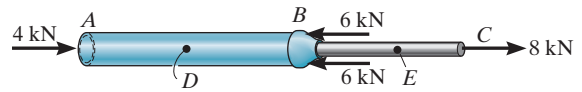
Ans.



Ans:

$$\sigma_{\text{avg}} = \frac{P}{A} \sin^2 \theta, \quad \tau_{\text{avg}} = \frac{P}{2A} \sin 2\theta$$

1-34. The built-up shaft consists of a pipe AB and solid rod BC . The pipe has an inner diameter of 20 mm and outer diameter of 28 mm. The rod has a diameter of 12 mm. Determine the average normal stress at points D and E and represent the stress on a volume element located at each of these points.



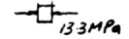
At D :

$$\sigma_D = \frac{P}{A} = \frac{4(10^3)}{\frac{\pi}{4}(0.028^2 - 0.02^2)} = 13.3 \text{ MPa (C)}$$

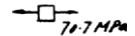
At E :

$$\sigma_E = \frac{P}{A} = \frac{8(10^3)}{\frac{\pi}{4}(0.012^2)} = 70.7 \text{ MPa (T)}$$

Ans.



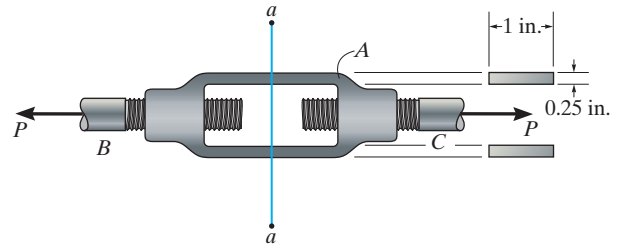
Ans.



Ans:

$$\sigma_D = 13.3 \text{ MPa (C)}, \sigma_E = 70.7 \text{ MPa (T)}$$

1-35. If the turnbuckle is subjected to an axial force of $P = 900$ lb, determine the average normal stress developed in section $a-a$ and in each of the bolt shanks at B and C . Each bolt shank has a diameter of 0.5 in.



Internal Loading: The normal force developed in section $a-a$ of the bracket and the bolt shank can be obtained by writing the force equations of equilibrium along the x axis with reference to the free-body diagrams of the sections shown in Figs. a and b , respectively.

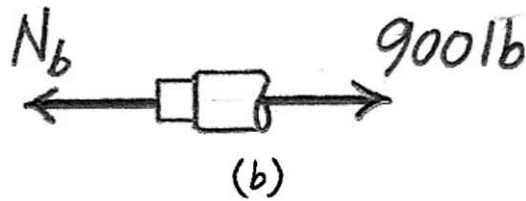
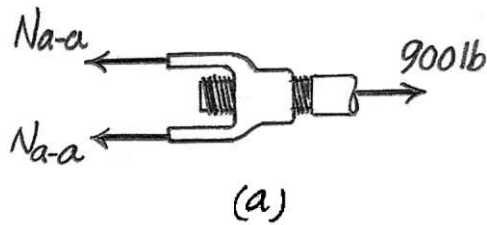
$$\begin{aligned} \rightarrow \Sigma F_x = 0; \quad 900 - 2N_{a-a} = 0 \quad N_{a-a} = 450 \text{ lb} \end{aligned}$$

$$\begin{aligned} \rightarrow \Sigma F_x = 0; \quad 900 - N_b = 0 \quad N_b = 900 \text{ lb} \end{aligned}$$

Average Normal Stress: The cross-sectional areas of section $a-a$ and the bolt shank are $A_{a-a} = (1)(0.25) = 0.25 \text{ in}^2$ and $A_b = \frac{\pi}{4}(0.5^2) = 0.1963 \text{ in}^2$, respectively. We obtain

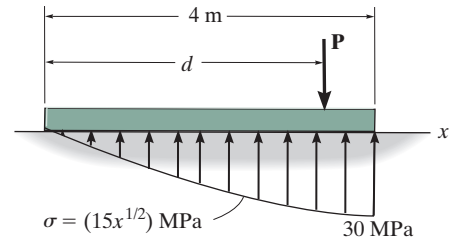
$$(\sigma_{a-a})_{\text{avg}} = \frac{N_{a-a}}{A_{a-a}} = \frac{450}{0.25} = 1800 \text{ psi} = 1.80 \text{ ksi} \quad \text{Ans.}$$

$$\sigma_b = \frac{N_b}{A_b} = \frac{900}{0.1963} = 4584 \text{ psi} = 4.58 \text{ ksi} \quad \text{Ans.}$$



Ans:
 $(\sigma_{a-a})_{\text{avg}} = 1.80 \text{ ksi}, \sigma_b = 4.58 \text{ ksi}$

1-37. The plate has a width of 0.5 m. If the stress distribution at the support varies as shown, determine the force **P** applied to the plate and the distance *d* to where it is applied.



The resultant force dF of the bearing pressure acting on the plate of area $dA = b dx = 0.5 dx$, Fig. *a*,

$$dF = \sigma_b dA = (15x^{1/2})(10^6)(0.5dx) = 7.5(10^6)x^{1/2} dx$$

$$+\uparrow \Sigma F_y = 0; \quad \int dF - P = 0$$

$$\int_0^{4\text{m}} 7.5(10^6)x^{1/2} dx - P = 0$$

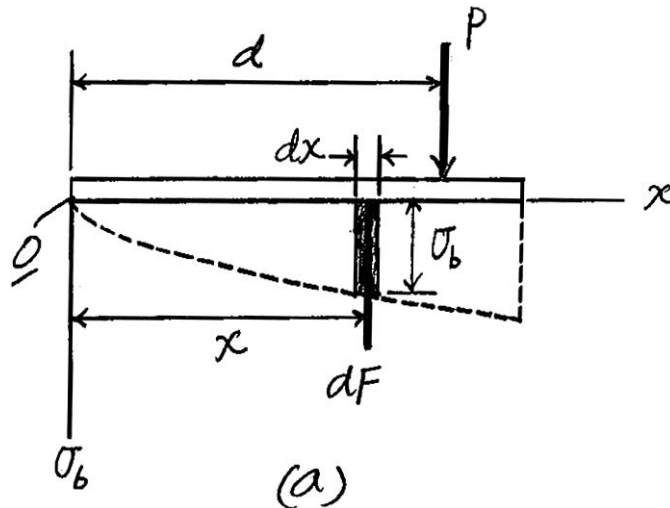
$$P = 40(10^6) \text{ N} = 40 \text{ MN} \quad \text{Ans.}$$

Equilibrium requires

$$\zeta + \Sigma M_O = 0; \quad \int x dF - Pd = 0$$

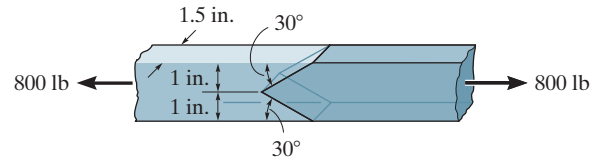
$$\int_0^{4\text{m}} x[7.5(10^6)x^{1/2} dx] - 40(10^6) d = 0$$

$$d = 2.40 \text{ m} \quad \text{Ans.}$$



Ans:
 $P = 40 \text{ MN}, d = 2.40 \text{ m}$

1-38. The two members used in the construction of an aircraft fuselage are joined together using a 30° fish-mouth weld. Determine the average normal and average shear stress on the plane of each weld. Assume each inclined plane supports a horizontal force of 400 lb.



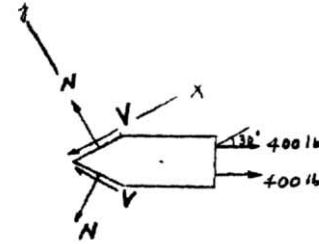
$$N - 400 \sin 30^\circ = 0; \quad N = 200 \text{ lb}$$

$$400 \cos 30^\circ - V = 0; \quad V = 346.41 \text{ lb}$$

$$A' = \frac{1.5(1)}{\sin 30^\circ} = 3 \text{ in}^2$$

$$\sigma = \frac{N}{A'} = \frac{200}{3} = 66.7 \text{ psi}$$

$$\tau = \frac{V}{A'} = \frac{346.41}{3} = 115 \text{ psi}$$



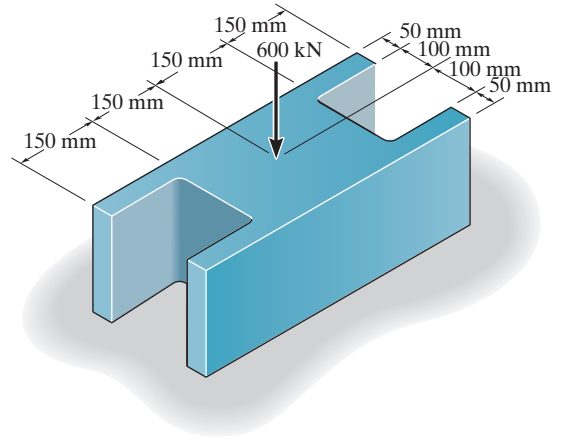
Ans.

Ans.

Ans:

$$\sigma = 66.7 \text{ psi}, \tau = 115 \text{ psi}$$

1-39. If the block is subjected to the centrally applied force of 600 kN, determine the average normal stress in the material. Show the stress acting on a differential volume element of the material.

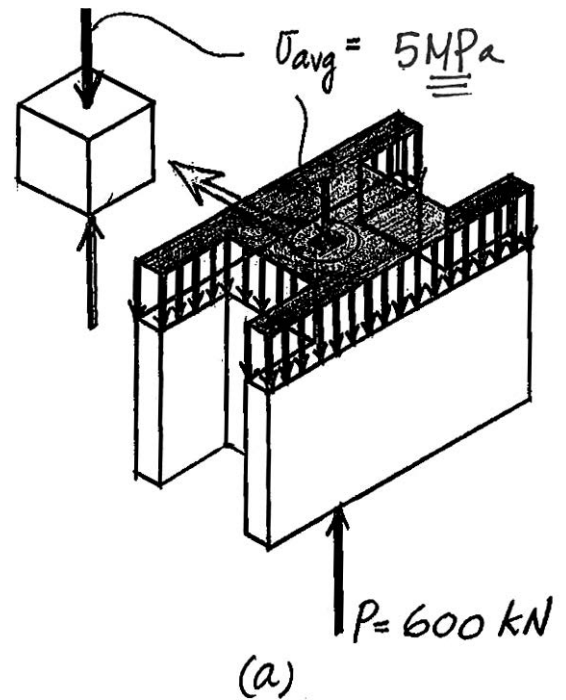


The cross-sectional area of the block is $A = 0.6(0.3) - 0.3(0.2) = 0.12 \text{ m}^2$.

$$\sigma_{\text{avg}} = \frac{P}{A} = \frac{600(10^3)}{0.12} = 5(10^6) \text{ Pa} = 5 \text{ MPa}$$

Ans.

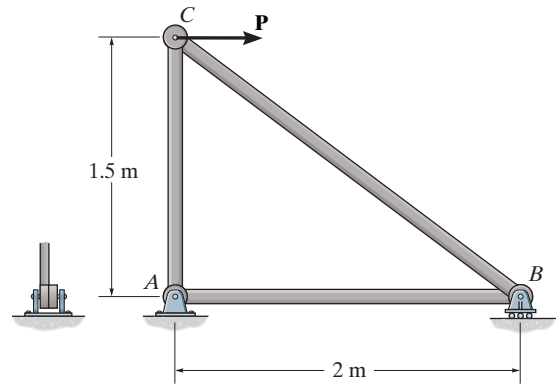
The average normal stress distribution over the cross section of the block and the state of stress of a point in the block represented by a differential volume element are shown in Fig. *a*



Ans:

$$\sigma_{\text{avg}} = 5 \text{ MPa}$$

***1-40.** Determine the average normal stress in each of the 20-mm diameter bars of the truss. Set $P = 40$ kN.



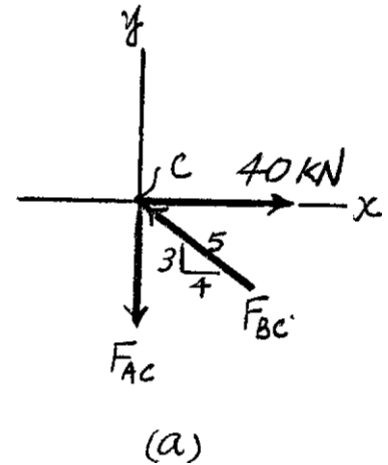
Internal Loadings: The force developed in each member of the truss can be determined by using the method of joints. First, consider the equilibrium of joint C, Fig. a,

$$\rightarrow \Sigma F_x = 0; \quad 40 - F_{BC} \left(\frac{4}{5} \right) = 0 \quad F_{BC} = 50 \text{ kN (C)}$$

$$+\uparrow \Sigma F_y = 0; \quad 50 \left(\frac{3}{5} \right) - F_{AC} = 0 \quad F_{AC} = 30 \text{ kN (T)}$$

Subsequently, the equilibrium of joint B, Fig. b, is considered

$$\rightarrow \Sigma F_x = 0; \quad 50 \left(\frac{4}{5} \right) - F_{AB} = 0 \quad F_{AB} = 40 \text{ kN (T)}$$



Average Normal Stress: The cross-sectional area of each of the bars is

$$A = \frac{\pi}{4} (0.02^2) = 0.3142(10^{-3}) \text{ m}^2. \text{ We obtain,}$$

$$(\sigma_{\text{avg}})_{BC} = \frac{F_{BC}}{A} = \frac{50(10^3)}{0.3142(10^{-3})} = 159 \text{ MPa}$$

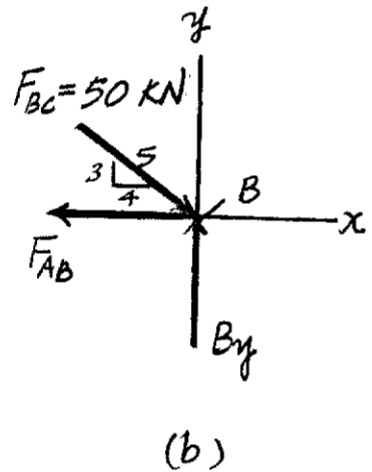
Ans.

$$(\sigma_{\text{avg}})_{AC} = \frac{F_{AC}}{A} = \frac{30(10^3)}{0.3142(10^{-3})} = 95.5 \text{ MPa}$$

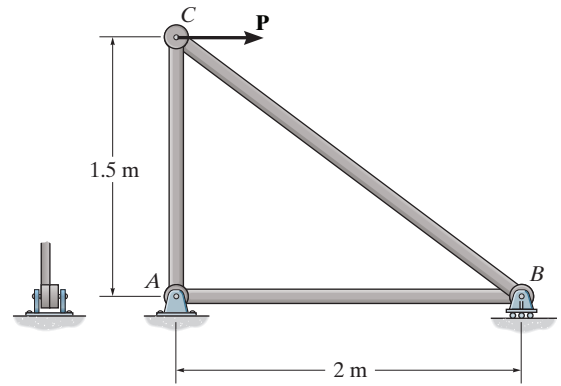
Ans.

$$(\sigma_{\text{avg}})_{AB} = \frac{F_{AB}}{A} = \frac{40(10^3)}{0.3142(10^{-3})} = 127 \text{ MPa}$$

Ans.



1-41. If the average normal stress in each of the 20-mm-diameter bars is not allowed to exceed 150 MPa, determine the maximum force **P** that can be applied to joint **C**.



Internal Loadings: The force developed in each member of the truss can be determined by using the method of joints. First, consider the equilibrium of joint **C**, Fig. *a*,

$$\rightarrow \Sigma F_x = 0; \quad P - F_{BC} \left(\frac{4}{5} \right) = 0 \quad F_{BC} = 1.25P(C)$$

$$+\uparrow \Sigma F_y = 0; \quad 1.25P \left(\frac{3}{5} \right) - F_{AC} = 0 \quad F_{AC} = 0.75P(T)$$

Subsequently, the equilibrium of joint **B**, Fig. *b*, is considered

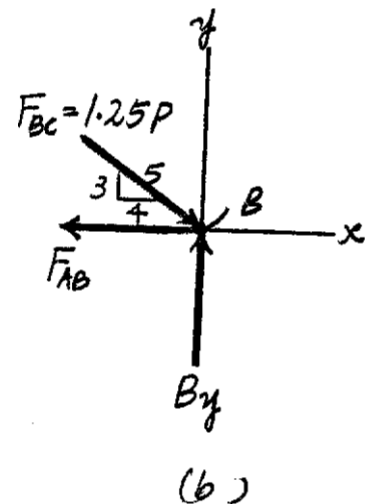
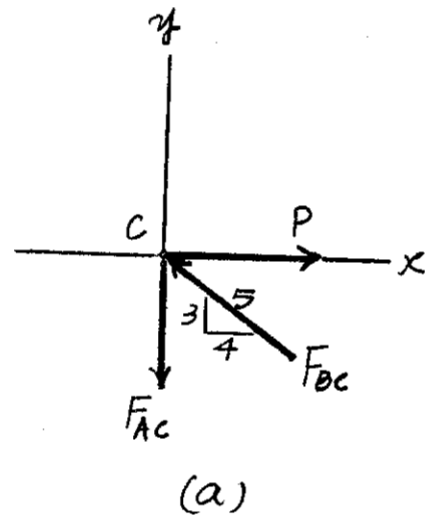
$$\rightarrow \Sigma F_x = 0; \quad 1.25P \left(\frac{4}{5} \right) - F_{AB} = 0 \quad F_{AB} = P(T)$$

Average Normal Stress: Since the cross-sectional area and the allowable normal stress of each bar are the same, member **BC** which is subjected to the maximum normal force is the critical member. The cross-sectional area of each of the bars is $A = \frac{\pi}{4} (0.02^2) = 0.3142(10^{-3}) \text{ m}^2$. We have,

$$(\sigma_{\text{avg}})_{\text{allow}} = \frac{F_{BC}}{A}; \quad 150(10^6) = \frac{1.25P}{0.3142(10^{-3})}$$

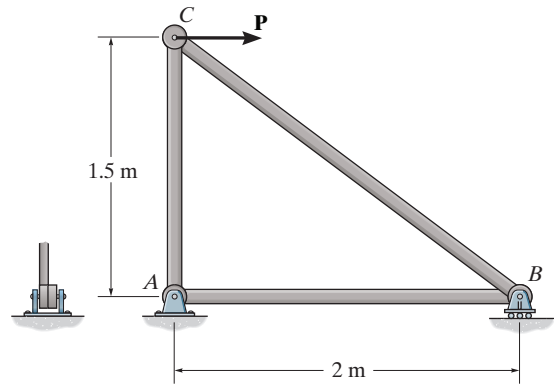
$$P = 37\,699 \text{ N} = 37.7 \text{ kN}$$

Ans.



Ans:
 $P = 37.7 \text{ kN}$

1-42. Determine the average shear stress developed in pin A of the truss. A horizontal force of $P = 40$ kN is applied to joint C . Each pin has a diameter of 25 mm and is subjected to double shear.



Internal Loadings: The forces acting on pins A and B are equal to the support reactions at A and B . Referring to the free-body diagram of the entire truss, Fig. a ,

$$\sum M_A = 0; \quad B_y(2) - 40(1.5) = 0 \quad B_y = 30 \text{ kN}$$

$$\rightarrow \sum F_x = 0; \quad 40 - A_x = 0 \quad A_x = 40 \text{ kN}$$

$$+\uparrow \sum F_y = 0; \quad 30 - A_y = 0 \quad A_y = 30 \text{ kN}$$

Thus,

$$F_A = \sqrt{A_x^2 + A_y^2} = \sqrt{40^2 + 30^2} = 50 \text{ kN}$$

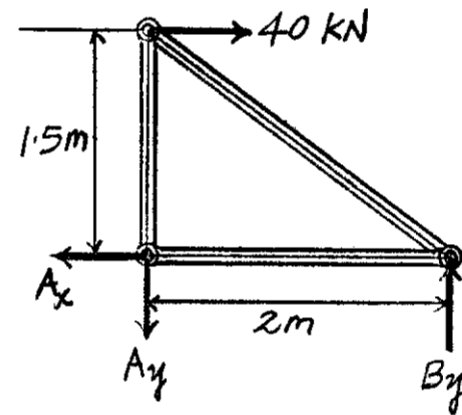
Since pin A is in double shear, Fig. b , the shear forces developed on the shear planes of pin A are

$$V_A = \frac{F_A}{2} = \frac{50}{2} = 25 \text{ kN}$$

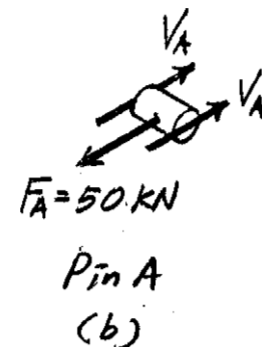
Average Shear Stress: The area of the shear plane for pin A is $A_A = \frac{\pi}{4}(0.025^2) = 0.4909(10^{-3}) \text{ m}^2$. We have

$$(\tau_{\text{avg}})_A = \frac{V_A}{A_A} = \frac{25(10^3)}{0.4909(10^{-3})} = 50.9 \text{ MPa}$$

Ans.

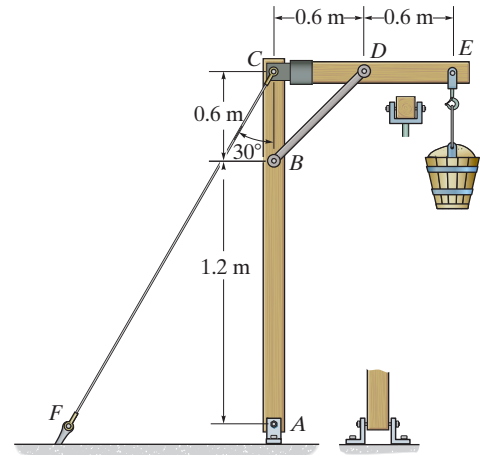


(a)



Ans:
 $(\tau_{\text{avg}})_A = 50.9 \text{ MPa}$

1-43. The 150-kg bucket is suspended from end E of the frame. Determine the average normal stress in the 6-mm diameter wire CF and the 15-mm diameter short strut BD .



Internal Loadings: The normal force developed in rod BD and cable CF can be determined by writing the moment equations of equilibrium about C and A with reference to the free-body diagram of member CE and the entire frame shown in Figs. a and b , respectively.

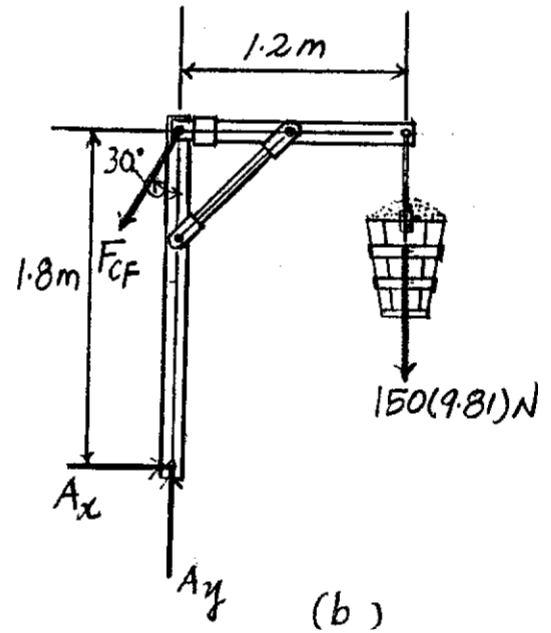
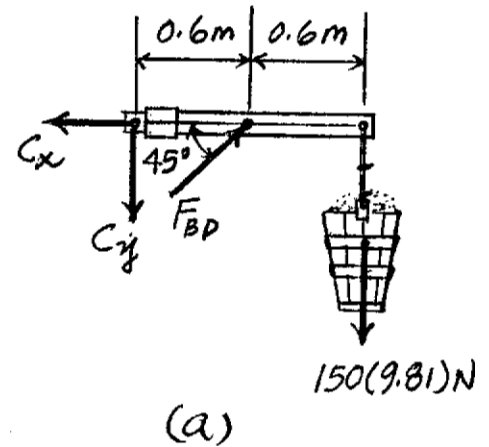
$$\zeta + \Sigma M_C = 0; \quad F_{BD} \sin 45^\circ(0.6) - 150(9.81)(1.2) = 0 \quad F_{BD} = 4162.03 \text{ N}$$

$$\zeta + \Sigma M_A = 0; \quad F_{CF} \sin 30^\circ(1.8) - 150(9.81)(1.2) = 0 \quad F_{CF} = 1962 \text{ N}$$

Average Normal Stress: The cross-sectional areas of rod BD and cable CF are $A_{BD} = \frac{\pi}{4}(0.015^2) = 0.1767(10^{-3}) \text{ m}^2$ and $A_{CF} = \frac{\pi}{4}(0.006^2) = 28.274(10^{-6}) \text{ m}^2$. We have

$$(\sigma_{\text{avg}})_{BD} = \frac{F_{BD}}{A_{BD}} = \frac{4162.03}{0.1767(10^{-3})} = 23.6 \text{ MPa}$$

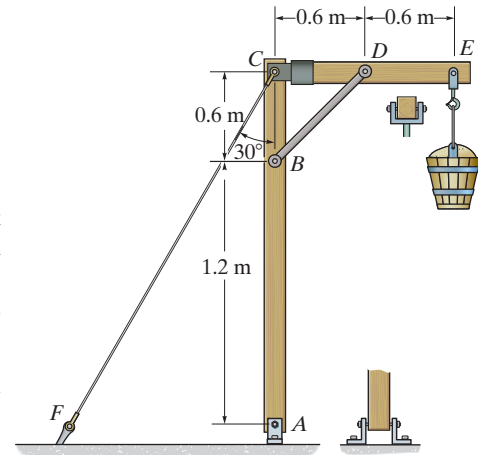
$$(\sigma_{\text{avg}})_{CF} = \frac{F_{CF}}{A_{CF}} = \frac{1962}{28.274(10^{-6})} = 69.4 \text{ MPa}$$



Ans:

$$(\sigma_{\text{avg}})_{BD} = 23.6 \text{ MPa}, (\sigma_{\text{avg}})_{CF} = 69.4 \text{ MPa}$$

*1-44. The 150-kg bucket is suspended from end E of the frame. If the diameters of the pins at A and D are 6 mm and 10 mm, respectively, determine the average shear stress developed in these pins. Each pin is subjected to double shear.



Internal Loading: The forces exerted on pins D and A are equal to the support reaction at D and A . First, consider the free-body diagram of member CE shown in Fig. a .

$$\zeta + \Sigma M_C = 0; \quad F_{BD} \sin 45^\circ(0.6) - 150(9.81)(1.2) = 0 \quad F_{BD} = 4162.03 \text{ N}$$

Subsequently, the free-body diagram of the entire frame shown in Fig. b will be considered.

$$\zeta + \Sigma M_A = 0; \quad F_{CF} \sin 30^\circ(1.8) - 150(9.81)(1.2) = 0 \quad F_{CF} = 1962 \text{ N}$$

$$\rightarrow \Sigma F_x = 0; \quad A_x - 1962 \sin 30^\circ = 0 \quad A_x = 981 \text{ N}$$

$$+\uparrow \Sigma F_y = 0; \quad A_y - 1962 \cos 30^\circ - 150(9.81) = 0 \quad A_y = 3170.64 \text{ N}$$

Thus, the forces acting on pins D and A are

$$F_D = F_{BD} = 4162.03 \text{ N} \quad F_A = \sqrt{A_x^2 + A_y^2} = \sqrt{981^2 + 3170.64^2} = 3318.93 \text{ N}$$

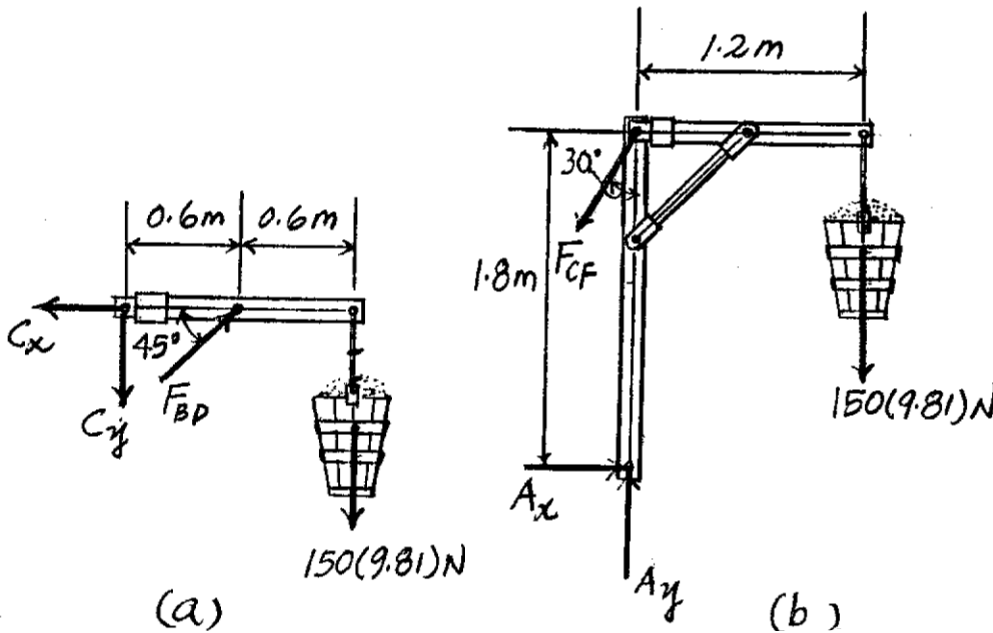
Since both pins are in double shear

$$V_D = \frac{F_D}{2} = 2081.02 \text{ N} \quad V_A = \frac{F_A}{2} = 1659.47 \text{ N}$$

Average Shear Stress: The cross-sectional areas of the shear plane of pins D and A are $A_D = \frac{\pi}{4}(0.01^2) = 78.540(10^{-6}) \text{ m}^2$ and $A_A = \frac{\pi}{4}(0.006^2) = 28.274(10^{-6}) \text{ m}^2$. We obtain

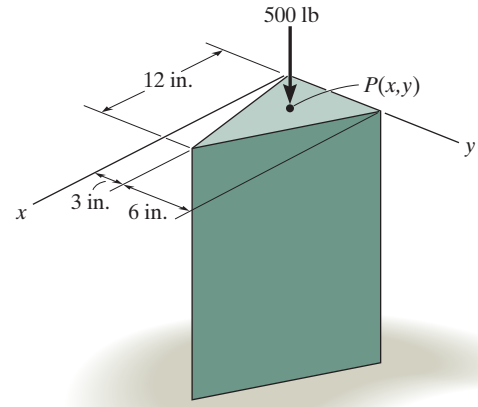
$$(\tau_{\text{avg}})_A = \frac{V_A}{A_A} = \frac{1659.47}{28.274(10^{-6})} = 58.7 \text{ MPa} \quad \text{Ans.}$$

$$(\tau_{\text{avg}})_D = \frac{V_D}{A_D} = \frac{2081.02}{78.540(10^{-6})} = 26.5 \text{ MPa} \quad \text{Ans.}$$



Ans:
 $x = 4 \text{ in.}, y = 4 \text{ in.}, \sigma = 9.26 \text{ psi}$

1-45. The pedestal has a triangular cross section as shown. If it is subjected to a compressive force of 500 lb, specify the x and y coordinates for the location of point $P(x, y)$, where the load must be applied on the cross section, so that the average normal stress is uniform. Compute the stress and sketch its distribution acting on the cross section at a location removed from the point of load application.



$$x = \frac{\frac{1}{2}(3)(12)\left(\frac{12}{3}\right) + \frac{1}{2}(6)(12)\left(\frac{12}{3}\right)}{\frac{1}{2}(9)(12)} = 4 \text{ in.}$$

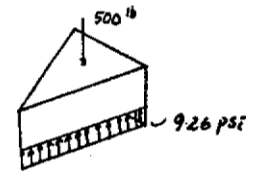
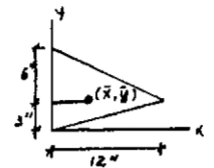
$$y = \frac{\frac{1}{2}(3)(12)(3)\left(\frac{2}{3}\right) + \frac{1}{2}(6)(12)\left(3 + \frac{6}{3}\right)}{\frac{1}{2}(9)(12)} = 4 \text{ in.}$$

$$\sigma = \frac{P}{A} = \frac{500}{\frac{1}{2}(9)(12)} = 9.26 \text{ psi}$$

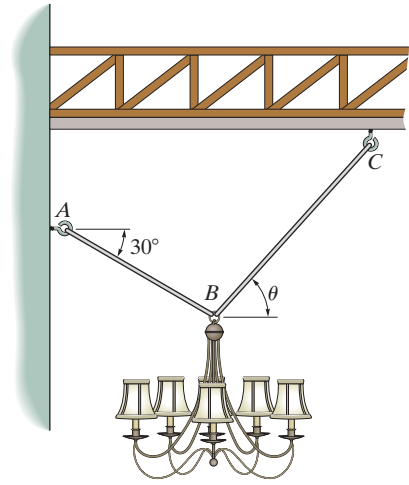
Ans.

Ans.

Ans.



1-46. The 20-kg chandelier is suspended from the wall and ceiling using rods AB and BC , which have diameters of 3 mm and 4 mm, respectively. Determine the angle θ so that the average normal stress in both rods is the same.



Internal Loadings: The force developed in cables AB and BC can be determined by considering the equilibrium of joint B , Fig. a ,

$$\rightarrow \Sigma F_x = 0; \quad F_{BC} \cos \theta - F_{AB} \cos 30^\circ = 0 \quad (1)$$

Average Normal Stress: The cross-sectional areas of cables AB and BC are $A_{AB} = \frac{\pi}{4}(0.003^2) = 7.069(10^{-6}) \text{ m}^2$ and $A_{BC} = \frac{\pi}{4}(0.004^2) = 12.566(10^{-6}) \text{ m}^2$. Since the average normal stress in both cables are required to be the same, then

$$\begin{aligned} (\sigma_{\text{avg}})_{AB} &= (\sigma_{\text{avg}})_{BC} \\ \frac{F_{AB}}{A_{AB}} &= \frac{F_{BC}}{A_{BC}} \\ \frac{F_{AB}}{7.069(10^{-6})} &= \frac{F_{BC}}{12.566(10^{-6})} \\ F_{AB} &= 0.5625 F_{BC} \end{aligned} \quad (2)$$

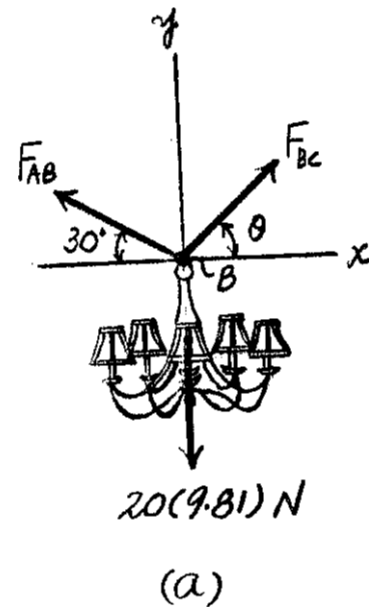
Substituting Eq. (2) into Eq. (1),

$$F_{BC}(\cos \theta - 0.5625 \cos 30^\circ) = 0$$

Since $F_{BC} \neq 0$, then

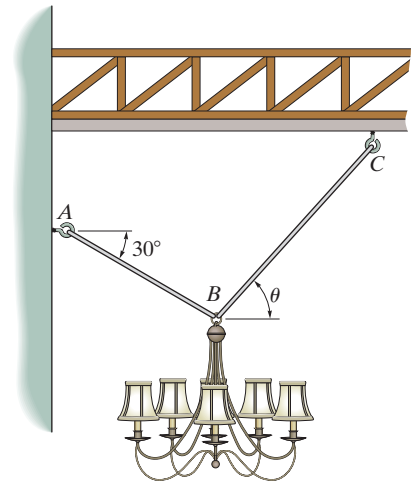
$$\begin{aligned} \cos \theta - 0.5625 \cos 30^\circ &= 0 \\ \theta &= 60.8^\circ \end{aligned}$$

Ans.



Ans:
 $\theta = 60.8^\circ$

1-47. The chandelier is suspended from the wall and ceiling using rods AB and BC , which have diameters of 3 mm and 4 mm, respectively. If the average normal stress in both rods is not allowed to exceed 150 MPa, determine the largest mass of the chandelier that can be supported if $\theta = 45^\circ$.



Internal Loadings: The force developed in cables AB and BC can be determined by considering the equilibrium of joint B , Fig. a ,

$$\rightarrow \Sigma F_x = 0; \quad F_{BC} \cos 45^\circ - F_{AB} \cos 30^\circ = 0 \quad (1)$$

$$+\uparrow \Sigma F_y = 0; \quad F_{BC} \sin 45^\circ + F_{AB} \sin 30^\circ - m(9.81) = 0 \quad (2)$$

Solving Eqs. (1) and (2) yields

$$F_{AB} = 7.181m \quad F_{BC} = 8.795m$$

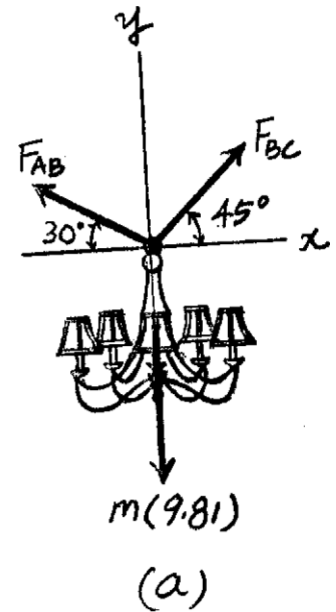
Average Normal Stress: The cross-sectional areas of cables AB and BC are $A_{AB} = \frac{\pi}{4}(0.003^2) = 7.069(10^{-6}) \text{ m}^2$ and $A_{BC} = \frac{\pi}{4}(0.004^2) = 12.566(10^{-6}) \text{ m}^2$. We have,

$$(\sigma_{\text{avg}})_{\text{allow}} = \frac{F_{AB}}{A_{AB}}; \quad 150(10^6) = \frac{7.181m}{7.069(10^{-6})}$$

$$m = 147.64 \text{ kg} = 148 \text{ kg (controls)} \quad \text{Ans.}$$

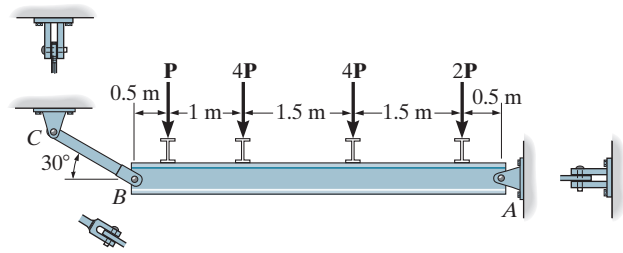
$$(\sigma_{\text{avg}})_{\text{allow}} = \frac{F_{BC}}{A_{BC}}; \quad 150(10^6) = \frac{8.795m}{12.566(10^{-6})}$$

$$m = 214.31 \text{ kg}$$



Ans:
 $m = 148 \text{ kg}$

*1-48. The beam is supported by a pin at A and a short link BC . If $P = 15$ kN, determine the average shear stress developed in the pins at A , B , and C . All pins are in double shear as shown, and each has a diameter of 18 mm.



For pins B and C :

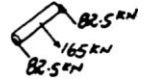
$$\tau_B = \tau_C = \frac{V}{A} = \frac{82.5 (10^3)}{\frac{\pi}{4} (18)^2} = 324 \text{ MPa}$$

For pin A :

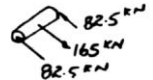
$$F_A = \sqrt{(82.5)^2 + (142.9)^2} = 165 \text{ kN}$$

$$\tau_A = \frac{V}{A} = \frac{82.5 (10^3)}{\frac{\pi}{4} (18)^2} = 324 \text{ MPa}$$

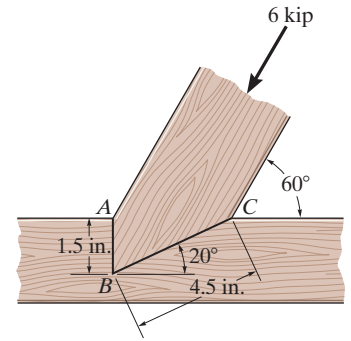
Ans.



Ans.



1-49. The joint is subjected to the axial member force of 6 kip. Determine the average normal stress acting on sections AB and BC . Assume the member is smooth and is 1.5-in. thick.



$$+\uparrow \Sigma F_y = 0; \quad -6 \sin 60^\circ + N_{BC} \cos 20^\circ = 0$$

$$N_{BC} = 5.530 \text{ kip}$$

$$\rightarrow \Sigma F_x = 0; \quad N_{AB} - 6 \cos 60^\circ - 5.530 \sin 20^\circ = 0$$

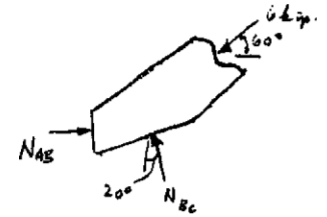
$$N_{AB} = 4.891 \text{ kip}$$

$$\sigma_{AB} = \frac{N_{AB}}{A_{AB}} = \frac{4.891}{(1.5)(1.5)} = 2.17 \text{ ksi}$$

Ans.

$$\sigma_{BC} = \frac{N_{BC}}{A_{BC}} = \frac{5.530}{(1.5)(4.5)} = 0.819 \text{ ksi}$$

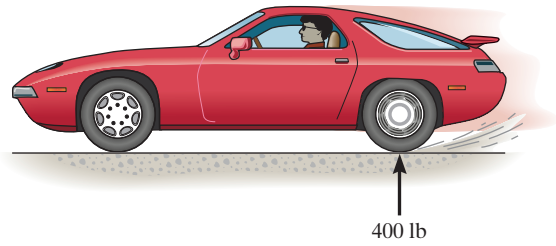
Ans.



Ans:

$$\sigma_{AB} = 2.17 \text{ ksi}, \sigma_{BC} = 0.819 \text{ ksi}$$

1-50. The driver of the sports car applies his rear brakes and causes the tires to slip. If the normal force on each rear tire is 400 lb and the coefficient of kinetic friction between the tires and the pavement is $\mu_k = 0.5$, determine the average shear stress developed by the friction force on the tires. Assume the rubber of the tires is flexible and each tire is filled with an air pressure of 32 psi.



$$F = \mu_k N = 0.5(400) = 200 \text{ lb}$$

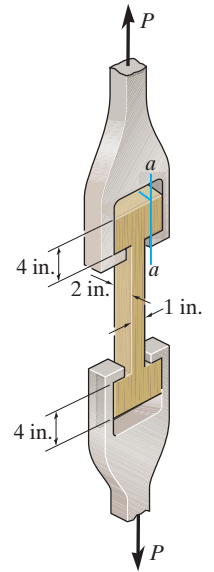
$$p = \frac{N}{A}; \quad A = \frac{400}{32} = 12.5 \text{ in}^2$$

$$\tau_{\text{avg}} = \frac{F}{A} = \frac{200}{12.5} = 16 \text{ psi}$$

Ans.

Ans:
 $\tau_{\text{avg}} = 16 \text{ psi}$

1-51. During the tension test, the wooden specimen is subjected to an average normal stress of 2 ksi. Determine the axial force **P** applied to the specimen. Also, find the average shear stress developed along section *a-a* of the specimen.



Internal Loading: The normal force developed on the cross section of the middle portion of the specimen can be obtained by considering the free-body diagram shown in Fig. *a*.

$$+\uparrow \Sigma F_y = 0; \quad \frac{P}{2} + \frac{P}{2} - N = 0 \quad N = P$$

Referring to the free-body diagram shown in fig. *b*, the shear force developed in the shear plane *a-a* is

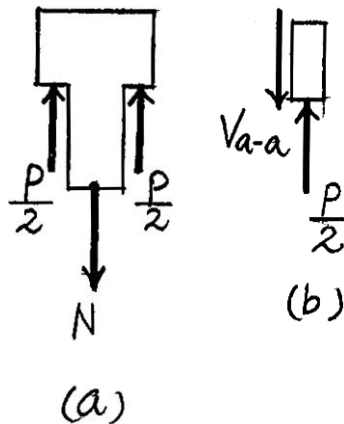
$$+\uparrow \Sigma F_y = 0; \quad \frac{P}{2} - V_{a-a} = 0 \quad V_{a-a} = \frac{P}{2}$$

Average Normal Stress and Shear Stress: The cross-sectional area of the specimen is $A = 1(2) = 2 \text{ in}^2$. We have

$$\begin{aligned} \sigma_{\text{avg}} &= \frac{N}{A}; & 2(10^3) &= \frac{P}{2} \\ & & P &= 4(10^3)\text{lb} = 4 \text{ kip} \end{aligned} \quad \text{Ans.}$$

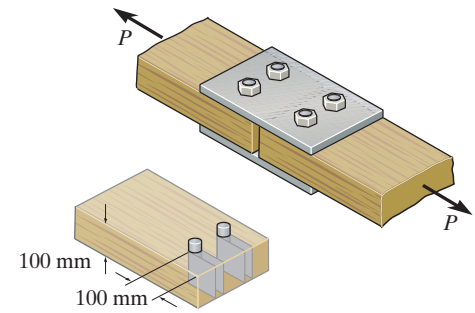
Using the result of **P**, $V_{a-a} = \frac{P}{2} = \frac{4(10^3)}{2} = 2(10^3) \text{ lb}$. The area of the shear plane is $A_{a-a} = 2(4) = 8 \text{ in}^2$. We obtain

$$(\tau_{a-a})_{\text{avg}} = \frac{V_{a-a}}{A_{a-a}} = \frac{2(10^3)}{8} = 250 \text{ psi} \quad \text{Ans.}$$



Ans:
 $P = 4 \text{ kip}, (\tau_{a-a})_{\text{avg}} = 250 \text{ psi}$

*1-52. If the joint is subjected to an axial force of $P = 9 \text{ kN}$, determine the average shear stress developed in each of the 6-mm diameter bolts between the plates and the members and along each of the four shaded shear planes.



Internal Loadings: The shear force developed on each shear plane of the bolt and the member can be determined by writing the force equation of equilibrium along the member's axis with reference to the free-body diagrams shown in Figs. *a*. and *b*, respectively.

$$\Sigma F_y = 0; \quad 4V_b - 9 = 0 \qquad V_b = 2.25 \text{ kN}$$

$$\Sigma F_y = 0; \quad 4V_p - 9 = 0 \qquad V_p = 2.25 \text{ kN}$$

Average Shear Stress: The areas of each shear plane of the bolt and the member are $A_b = \frac{\pi}{4}(0.006^2) = 28.274(10^{-6}) \text{ m}^2$ and $A_p = 0.1(0.1) = 0.01 \text{ m}^2$, respectively.

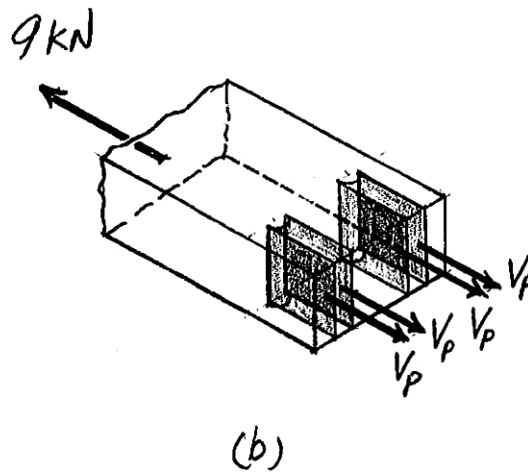
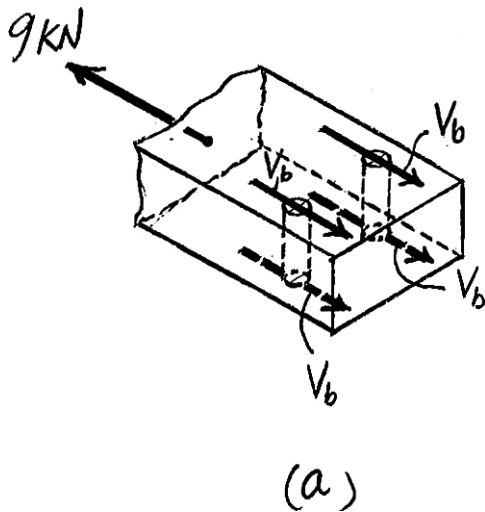
We obtain

$$(\tau_{\text{avg}})_b = \frac{V_b}{A_b} = \frac{2.25(10^3)}{28.274(10^{-6})} = 79.6 \text{ MPa}$$

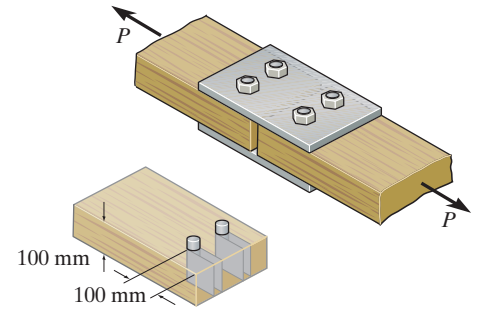
Ans.

$$(\tau_{\text{avg}})_p = \frac{V_p}{A_p} = \frac{2.25(10^3)}{0.01} = 225 \text{ kPa}$$

Ans.



1-53. The average shear stress in each of the 6-mm diameter bolts and along each of the four shaded shear planes is not allowed to exceed 80 MPa and 500 kPa, respectively. Determine the maximum axial force **P** that can be applied to the joint.



Internal Loadings: The shear force developed on each shear plane of the bolt and the member can be determined by writing the force equation of equilibrium along the member's axis with reference to the free-body diagrams shown in Figs. *a.* and *b.*, respectively.

$$\Sigma F_y = 0; \quad 4V_b - P = 0 \quad V_b = P/4$$

$$\Sigma F_y = 0; \quad 4V_p - P = 0 \quad V_p = P/4$$

Average Shear Stress: The areas of each shear plane of the bolts and the members are $A_b = \frac{\pi}{4} (0.006^2) = 28.274(10^{-6}) \text{ m}^2$ and $A_p = 0.1(0.1) = 0.01 \text{ m}^2$, respectively.

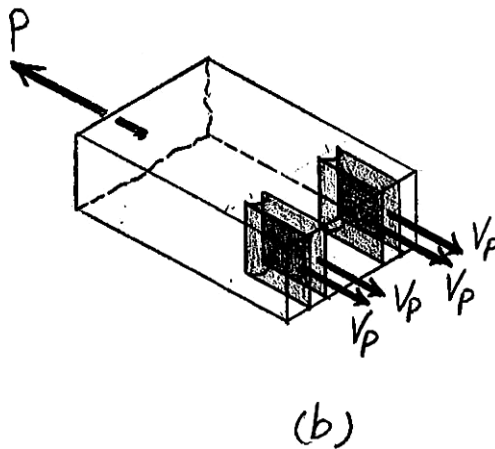
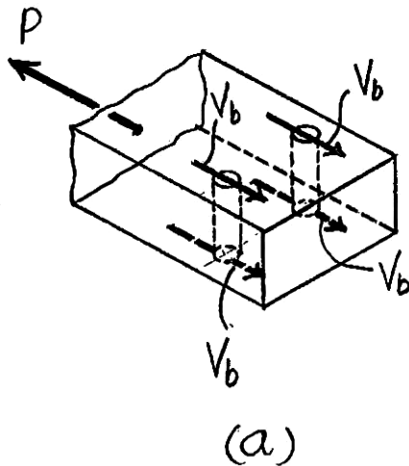
We obtain

$$(\tau_{\text{allow}})_b = \frac{V_b}{A_b}; \quad 80(10^6) = \frac{P/4}{28.274(10^{-6})}$$

$$P = 9047 \text{ N} = 9.05 \text{ kN (controls)} \quad \text{Ans.}$$

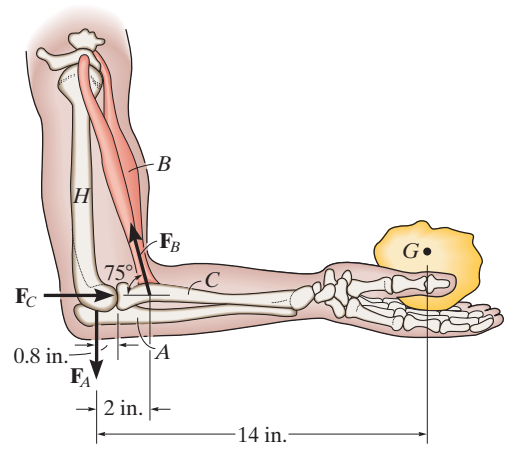
$$(\tau_{\text{allow}})_p = \frac{V_p}{A_p}; \quad 500(10^3) = \frac{P/4}{0.01}$$

$$P = 20\,000 \text{ N} = 20 \text{ kN}$$



Ans:
 $P = 9.05 \text{ kN}$

1-54. When the hand is holding the 5-lb stone, the humerus H , assumed to be smooth, exerts normal forces F_C and F_A on the radius C and ulna A , respectively, as shown. If the smallest cross-sectional area of the ligament at B is 0.30 in^2 , determine the greatest average tensile stress to which it is subjected.

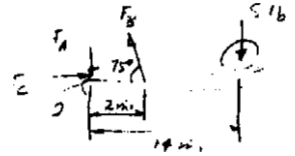


$$\zeta + \Sigma M_O = 0; \quad F_B \sin 75^\circ(2) - 5(14) = 0$$

$$F_B = 36.235 \text{ lb}$$

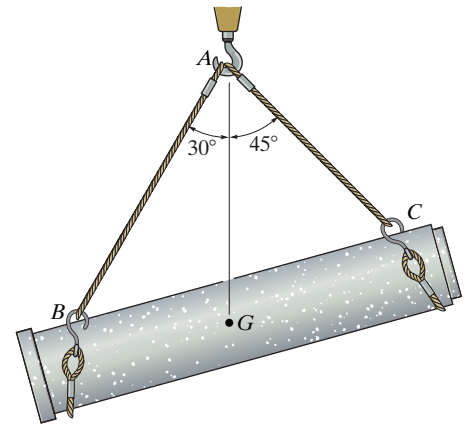
$$\sigma = \frac{P}{A} = \frac{36.235}{0.30} = 121 \text{ psi}$$

Ans.



Ans:
 $\sigma = 121 \text{ psi}$

1-55. The 2-Mg concrete pipe has a center of mass at point G . If it is suspended from cables AB and AC , determine the average normal stress developed in the cables. The diameters of AB and AC are 12 mm and 10 mm, respectively.



Internal Loadings: The normal force developed in cables AB and AC can be determined by considering the equilibrium of the hook for which the free-body diagram is shown in Fig. a .

$$\Sigma F_{x'} = 0; \quad 2000(9.81) \cos 45^\circ - F_{AB} \cos 15^\circ = 0 \quad F_{AB} = 14\,362.83 \text{ N (T)}$$

$$\Sigma F_{y'} = 0; \quad 2000(9.81) \sin 45^\circ - 14\,362.83 \sin 15^\circ - F_{AC} = 0 \quad F_{AC} = 10\,156.06 \text{ N (T)}$$

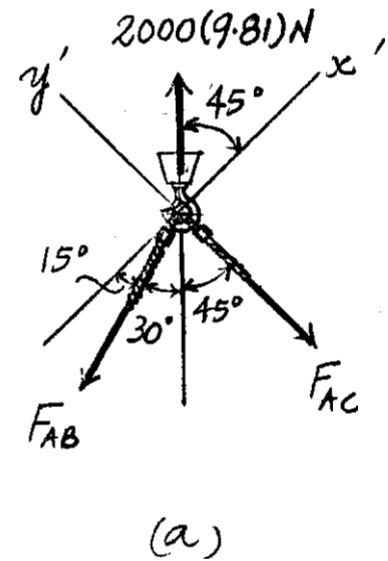
Average Normal Stress: The cross-sectional areas of cables AB and AC are $A_{AB} = \frac{\pi}{4}(0.012^2) = 0.1131(10^{-3}) \text{ m}^2$ and $A_{AC} = \frac{\pi}{4}(0.01^2) = 78.540(10^{-6}) \text{ m}^2$. We have,

$$\sigma_{AB} = \frac{F_{AB}}{A_{AB}} = \frac{14\,362.83}{0.1131(10^{-3})} = 127 \text{ MPa}$$

Ans.

$$\sigma_{AC} = \frac{F_{AC}}{A_{AC}} = \frac{10\,156.06}{78.540(10^{-6})} = 129 \text{ MPa}$$

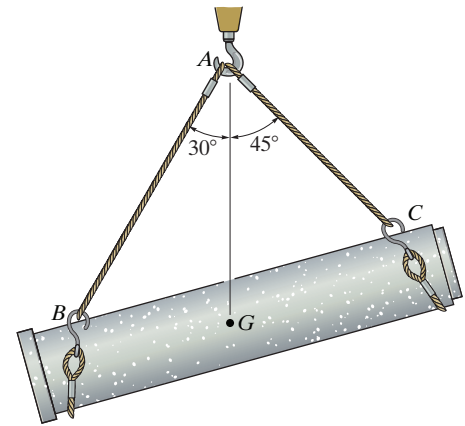
Ans.



Ans:

$$\sigma_{AB} = 127 \text{ MPa}, \sigma_{AC} = 129 \text{ MPa}$$

***1-56.** The 2-Mg concrete pipe has a center of mass at point G . If it is suspended from cables AB and AC , determine the diameter of cable AB so that the average normal stress developed in this cable is the same as in the 10-mm diameter cable AC .



Internal Loadings: The normal force in cables AB and AC can be determined by considering the equilibrium of the hook for which the free-body diagram is shown in Fig. a .

$$\Sigma F_{x'} = 0; 2000(9.81) \cos 45^\circ - F_{AB} \cos 15^\circ = 0 \quad F_{AB} = 14\,362.83 \text{ N (T)}$$

$$\Sigma F_{y'} = 0; 2000(9.81) \sin 45^\circ - 14\,362.83 \sin 15^\circ - F_{AC} = 0 \quad F_{AC} = 10\,156.06 \text{ N (T)}$$

Average Normal Stress: The cross-sectional areas of cables AB and AC are $A_{AB} = \frac{\pi}{4}d_{AB}^2$ and $A_{AC} = \frac{\pi}{4}(0.01^2) = 78.540(10^{-6}) \text{ m}^2$.

Here, we require

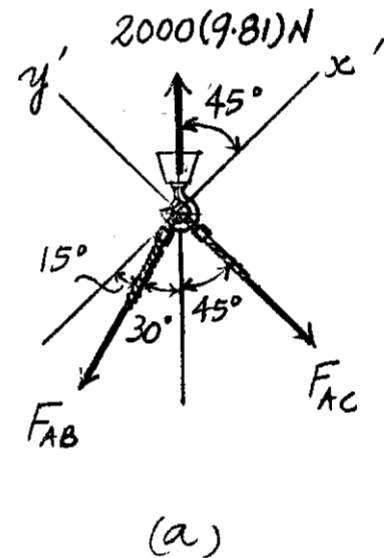
$$\sigma_{AB} = \sigma_{AC}$$

$$\frac{F_{AB}}{A_{AB}} = \frac{F_{AC}}{A_{AC}}$$

$$\frac{14\,362.83}{\frac{\pi}{4}d_{AB}^2} = \frac{10\,156.06}{78.540(10^{-6})}$$

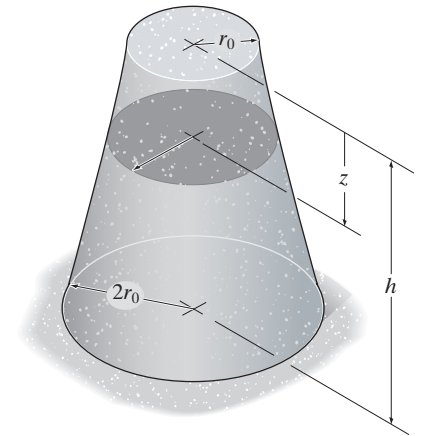
$$d_{AB} = 0.01189 \text{ m} = 11.9 \text{ mm}$$

Ans.



(a)

1-57. If the concrete pedestal has a specific weight of γ , determine the average normal stress developed in the pedestal as a function of z .



Internal Loading: From the geometry shown in Fig. a,

$$\frac{h'}{r_0} = \frac{h' + h}{2r_0}, \quad h' = h$$

and then

$$\frac{r}{z + h} = \frac{r_0}{h}, \quad r = \frac{r_0}{h}(z + h)$$

Thus, the volume of the frustum shown in Fig. b is

$$\begin{aligned} V &= \frac{1}{3} \left\{ \pi \left[\frac{r_0}{h}(z + h) \right]^2 \right\} (z + h) - \frac{1}{3} (\pi r_0^2) h \\ &= \frac{\pi r_0^2}{3h^2} [(z + h)^3 - h^3] \end{aligned}$$

The weight of this frustum is

$$W = \gamma V = \frac{\pi r_0^2 \gamma}{3h^2} [(z + h)^3 - h^3]$$

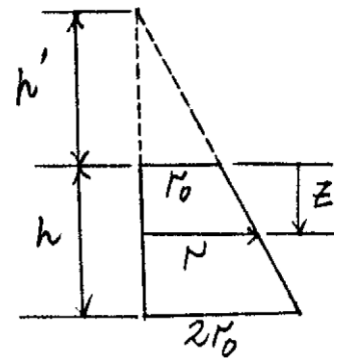
Average Normal Stress: The cross-sectional area the frustum as a function of z is

$$A = \pi r^2 = \frac{\pi r_0^2}{h^2} (z + h)^2.$$

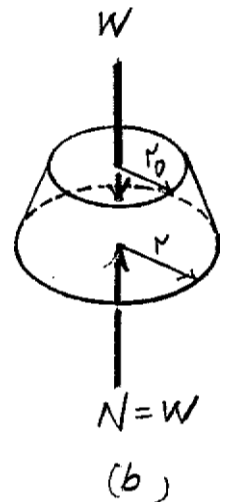
Also, the normal force acting on this cross section is $N = W$, Fig. b. We have

$$\sigma_{\text{avg}} = \frac{N}{A} = \frac{\frac{\pi r_0^2 \gamma}{3h^2} [(z + h)^3 - h^3]}{\frac{\pi r_0^2}{h^2} (z + h)^2} = \frac{\gamma [(z + h)^3 - h^3]}{3 [(z + h)^2]}$$

Ans.



(a)

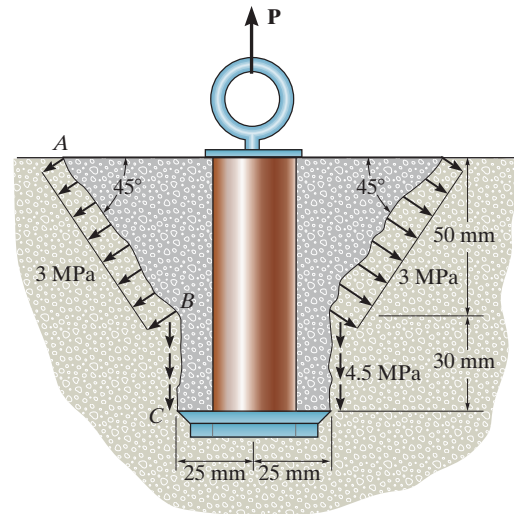


(b)

Ans:

$$\sigma_{\text{avg}} = \frac{\gamma [(z + h)^3 - h^3]}{3 [(z + h)^2]}$$

1-58. The anchor bolt was pulled out of the concrete wall and the failure surface formed part of a frustum and cylinder. This indicates a shear failure occurred along the cylinder BC and tension failure along the cylinder AB . If the shear and normal stresses along these surfaces have the magnitudes shown, determine the force P that must have been applied to the bolt.



Average Normal Stress:

For the frustum, $A = 2\pi\bar{x}L = 2\pi(0.025 + 0.025)(\sqrt{0.05^2 + 0.05^2})$

$$= 0.02221 \text{ m}^2$$

$$\sigma = \frac{P}{A}; \quad 3(10^6) = \frac{F_1}{0.02221}$$

$$F_1 = 66.64 \text{ kN}$$

Average Shear Stress:

For the cylinder, $A = \pi(0.05)(0.03) = 0.004712 \text{ m}^2$

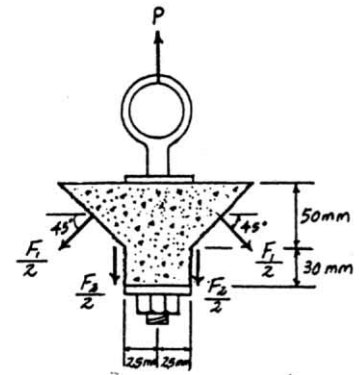
$$\tau_{\text{avg}} = \frac{V}{A}; \quad 4.5(10^6) = \frac{F_2}{0.004712}$$

$$F_2 = 21.21 \text{ kN}$$

Equation of Equilibrium:

$$+\uparrow \Sigma F_y = 0; \quad P - 21.21 - 66.64 \sin 45^\circ = 0$$

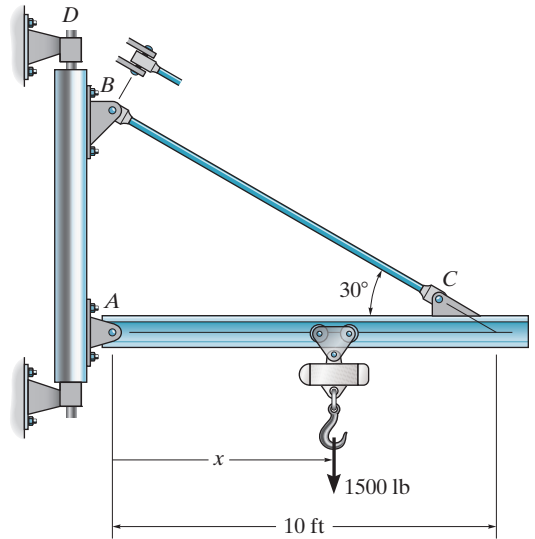
$$P = 68.3 \text{ kN}$$



Ans.

Ans:
 $P = 68.3 \text{ kN}$

1-59. The jib crane is pinned at A and supports a chain hoist that can travel along the bottom flange of the beam, $1 \text{ ft} \leq x \leq 12 \text{ ft}$. If the hoist is rated to support a maximum of 1500 lb , determine the maximum average normal stress in the $\frac{3}{4}$ -in. diameter tie rod BC and the maximum average shear stress in the $\frac{5}{8}$ -in. -diameter pin at B .



$$\zeta + \Sigma M_A = 0; \quad T_{BC} \sin 30^\circ(10) - 1500(x) = 0$$

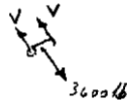
Maximum T_{BC} occurs when $x = 12 \text{ ft}$

$$T_{BC} = 3600 \text{ lb}$$

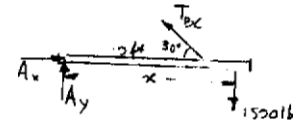
$$\sigma = \frac{P}{A} = \frac{3600}{\frac{\pi}{4}(0.75)^2} = 8.15 \text{ ksi}$$

$$\tau = \frac{V}{A} = \frac{3600/2}{\frac{\pi}{4}(5/8)^2} = 5.87 \text{ ksi}$$

Ans.



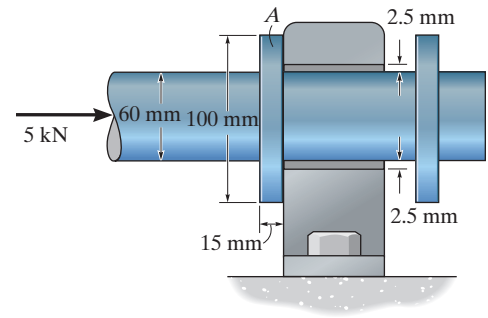
Ans.



Ans:

$$\sigma = 8.15 \text{ ksi}, \tau = 5.87 \text{ ksi}$$

*1-60. If the shaft is subjected to an axial force of 5 kN, determine the bearing stress acting on the collar A.

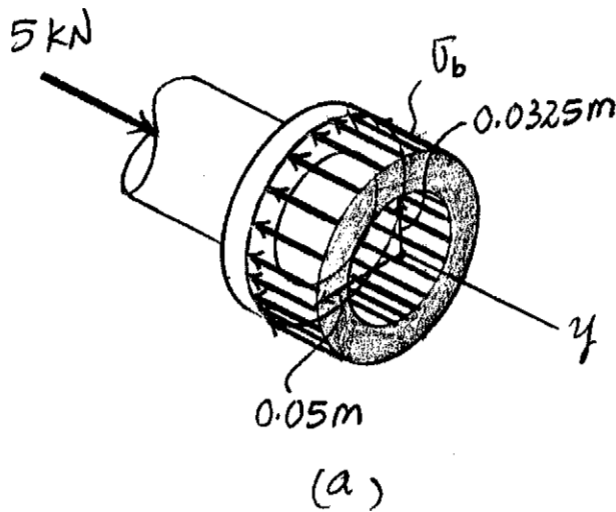


Bearing Stress: The bearing area on the collar, shown shaded in Fig. *a*, is $A_b = \pi(0.05^2 - 0.0325^2) = 4.536(10^{-3}) \text{ m}^2$. Referring to the free-body diagram of the collar, Fig. *a*, and writing the force equation of equilibrium along the axis of the shaft,

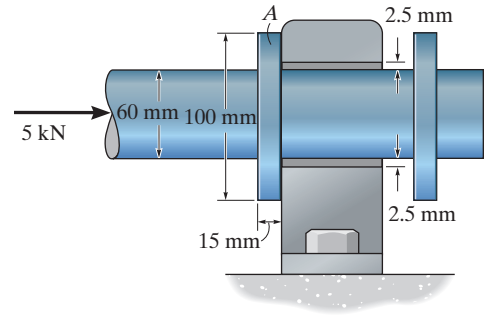
$$\Sigma F_y = 0; \quad 5(10^3) - \sigma_b [4.536(10^{-3})] = 0$$

$$\sigma_b = 1.10 \text{ MPa}$$

Ans.



1-61. If the 60-mm diameter shaft is subjected to an axial force of 5 kN, determine the average shear stress developed in the shear plane where the collar *A* and shaft are connected.

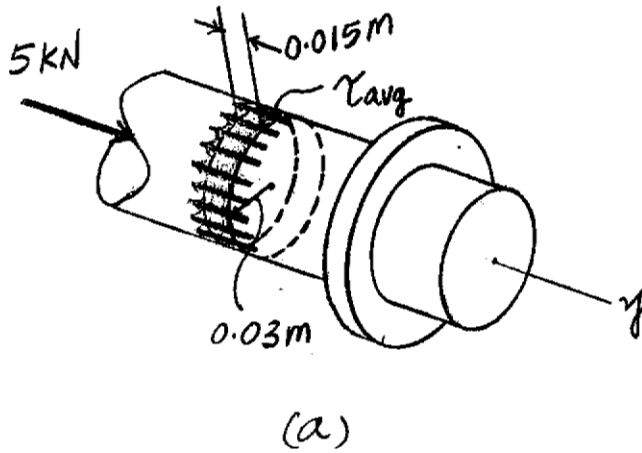


Average Shear Stress: The area of the shear plane, shown shaded in Fig. *a*, is $A = 2\pi(0.03)(0.015) = 2.827(10^{-3})\text{m}^2$. Referring to the free-body diagram of the shaft, Fig. *a*, and writing the force equation of equilibrium along the axis of the shaft,

$$\Sigma F_y = 0; 5(10^3) - \tau_{\text{avg}}[2.827(10^{-3})] = 0$$

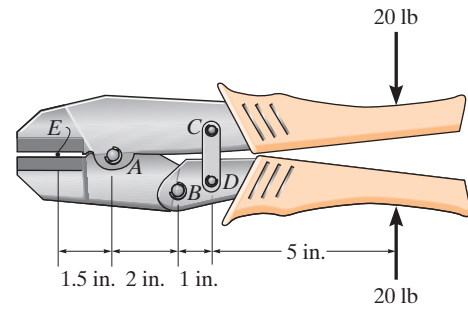
$$\tau_{\text{avg}} = 1.77 \text{ MPa}$$

Ans.



Ans:
 $\tau_{\text{avg}} = 1.77 \text{ MPa}$

1-62. The crimping tool is used to crimp the end of the wire *E*. If a force of 20 lb is applied to the handles, determine the average shear stress in the pin at *A*. The pin is subjected to double shear and has a diameter of 0.2 in. Only a vertical force is exerted on the wire.



Support Reactions:

From FBD(a)

$$\zeta + \Sigma M_D = 0; \quad 20(5) - B_y(1) = 0 \quad B_y = 100 \text{ lb}$$

$$\rightarrow \Sigma F_x = 0; \quad B_x = 0$$

From FBD(b)

$$\rightarrow \Sigma F_x = 0; \quad A_x = 0$$

$$\zeta + \Sigma M_E = 0; \quad A_y(1.5) - 100(3.5) = 0$$

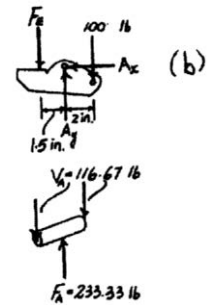
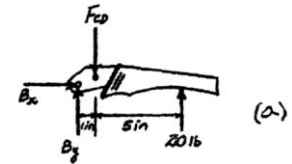
$$A_y = 233.33 \text{ lb}$$

Average Shear Stress: Pin *A* is subjected to double shear. Hence,

$$V_A = \frac{F_A}{2} = \frac{A_y}{2} = 116.67 \text{ lb}$$

$$(\tau_A)_{\text{avg}} = \frac{V_A}{A_A} = \frac{116.67}{\frac{\pi}{4}(0.2^2)}$$

$$= 3714 \text{ psi} = 3.71 \text{ ksi}$$



Ans.

Ans:
 $(\tau_A)_{\text{avg}} = 3.71 \text{ ksi}$

1-63. Solve Prob. 1-62 for pin B . The pin is subjected to double shear and has a diameter of 0.2 in.

Support Reactions:

From FBD(a)

$$\curvearrowleft + \sum M_D = 0; \quad 20(5) - B_y(1) = 0 \quad B_y = 100 \text{ lb}$$

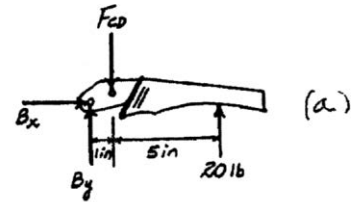
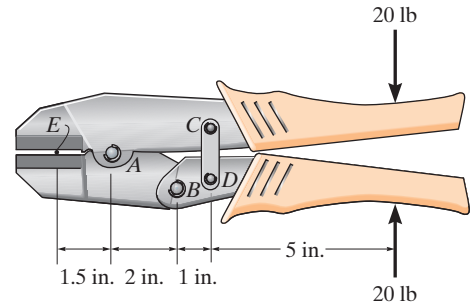
$$\rightarrow \sum F_x = 0; \quad B_x = 0$$

Average Shear Stress: Pin B is subjected to double shear. Hence,

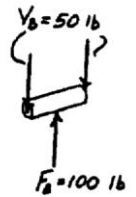
$$V_B = \frac{F_B}{2} = \frac{B_y}{2} = 50.0 \text{ lb}$$

$$(\tau_B)_{\text{avg}} = \frac{V_B}{A_B} = \frac{50.0}{\frac{\pi}{4}(0.2^2)}$$

$$= 1592 \text{ psi} = 1.59 \text{ ksi}$$

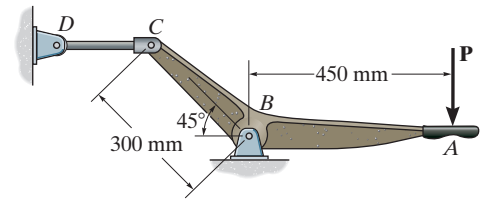


Ans.



Ans:
 $(\tau_B)_{\text{avg}} = 1.59 \text{ ksi}$

*1-64. A vertical force of $P = 1500\text{ N}$ is applied to the bell crank. Determine the average normal stress developed in the 10-mm diameter rod CD , and the average shear stress developed in the 6-mm diameter pin B that is subjected to double shear.



Internal Loading: Referring to the free-body diagram of the bell crank shown in Fig. *a*,

$$\begin{aligned} \zeta + \sum M_B = 0; & \quad F_{CD}(0.3 \sin 45^\circ) - 1500(0.45) = 0 & \quad F_{CD} = 3181.98\text{ N} \\ \rightarrow \sum F_x = 0; & \quad B_x - 3181.98 = 0 & \quad B_x = 3181.98\text{ N} \\ + \uparrow \sum F_y = 0; & \quad B_y - 1500 = 0 & \quad B_y = 1500\text{ N} \end{aligned}$$

Thus, the force acting on pin B is

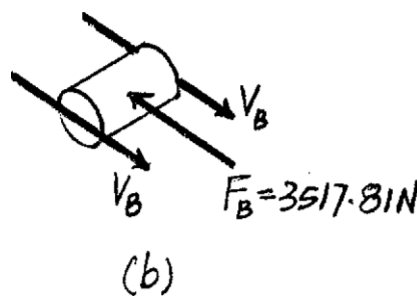
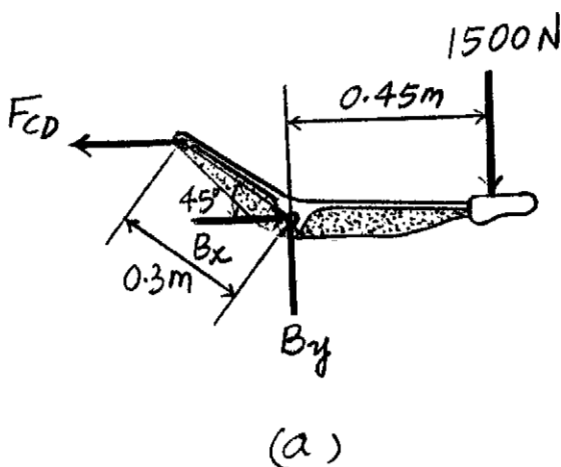
$$F_B = \sqrt{B_x^2 + B_y^2} = \sqrt{3181.98^2 + 1500^2} = 3517.81\text{ N}$$

Pin B is in double shear. Referring to its free-body diagram, Fig. *b*,

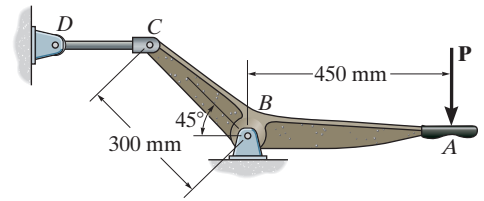
$$V_B = \frac{F_B}{2} = \frac{3517.81}{2} = 1758.91\text{ N}$$

Average Normal and Shear Stress: The cross-sectional area of rod CD is $A_{CD} = \frac{\pi}{4}(0.01^2) = 78.540(10^{-6})\text{ m}^2$, and the area of the shear plane of pin B is $A_B = \frac{\pi}{4}(0.006^2) = 28.274(10^{-6})\text{ m}^2$. We obtain

$$\begin{aligned} (\sigma_{\text{avg}})_{CD} &= \frac{F_{CD}}{A_{CD}} = \frac{3181.98}{78.540(10^{-6})} = 40.5\text{ MPa} & \quad \text{Ans.} \\ (\tau_{\text{avg}})_B &= \frac{V_B}{A_B} = \frac{1758.91}{28.274(10^{-6})} = 62.2\text{ MPa} & \quad \text{Ans.} \end{aligned}$$



1-65. Determine the maximum vertical force P that can be applied to the bell crank so that the average normal stress developed in the 10-mm diameter rod CD , and the average shear stress developed in the 6-mm diameter double shear pin B not exceed 175 MPa and 75 MPa, respectively.



Internal Loading: Referring to the free-body diagram of the bell crank shown in Fig. *a*,

$$\begin{aligned} \curvearrowright + \Sigma M_B = 0; & \quad F_{CD}(0.3 \sin 45^\circ) - P(0.45) = 0 & \quad F_{CD} = 2.121P \\ \rightarrow \Sigma F_x = 0; & \quad B_x - 2.121P = 0 & \quad B_x = 2.121P \\ +\uparrow \Sigma F_y = 0; & \quad B_y - P = 0 & \quad B_y = P \end{aligned}$$

Thus, the force acting on pin B is

$$F_B = \sqrt{B_x^2 + B_y^2} = \sqrt{(2.121P)^2 + P^2} = 2.345P$$

Pin B is in double shear. Referring to its free-body diagram, Fig. *b*,

$$V_B = \frac{F_B}{2} = \frac{2.345P}{2} = 1.173P$$

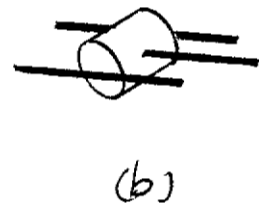
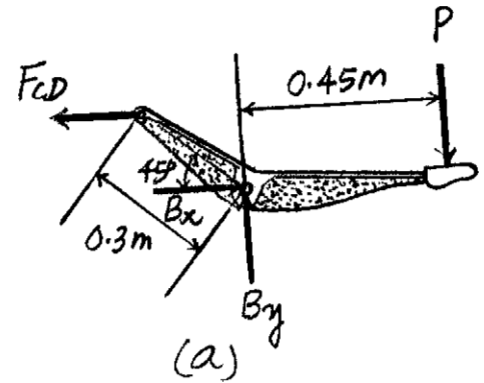
Average Normal and Shear Stress: The cross-sectional area of rod CD is

$$A_{CD} = \frac{\pi}{4}(0.01^2) = 78.540(10^{-6}) \text{ m}^2, \text{ and the area of the shear plane of pin } B$$

is $A_B = \frac{\pi}{4}(0.006^2) = 28.274(10^{-6}) \text{ m}^2$. We obtain

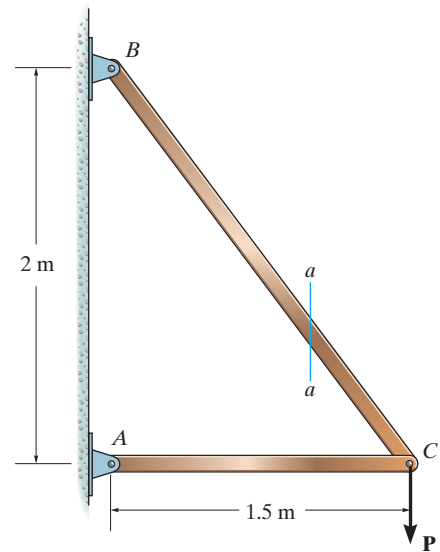
$$\begin{aligned} (\sigma_{\text{avg}})_{\text{allow}} = \frac{F_{CD}}{A_{CD}}; & \quad 175(10^6) = \frac{2.121P}{78.540(10^{-6})} \\ & \quad P = 6479.20 \text{ N} = 6.48 \text{ kN} \end{aligned}$$

$$\begin{aligned} (\tau_{\text{avg}})_{\text{allow}} = \frac{V_B}{A_B}; & \quad 75(10^6) = \frac{1.173P}{28.274(10^{-6})} \\ & \quad P = 1808.43 \text{ N} = 1.81 \text{ kN (controls)} \end{aligned} \quad \text{Ans.}$$



Ans:
 $P = 1.81 \text{ kN}$

1-66. Determine the largest load **P** that can be applied to the frame without causing either the average normal stress or the average shear stress at section *a-a* to exceed $\sigma = 150 \text{ MPa}$ and $\tau = 60 \text{ MPa}$, respectively. Member *CB* has a square cross section of 25 mm on each side.



Analyze the equilibrium of joint *C* using the FBD Shown in Fig. *a*,

$$+\uparrow \Sigma F_y = 0; \quad F_{BC} \left(\frac{4}{5} \right) - P = 0 \quad F_{BC} = 1.25P$$

Referring to the FBD of the cut segment of member *BC* Fig. *b*.

$$\rightarrow \Sigma F_x = 0; \quad N_{a-a} - 1.25P \left(\frac{3}{5} \right) = 0 \quad N_{a-a} = 0.75P$$

$$+\uparrow \Sigma F_y = 0; \quad 1.25P \left(\frac{4}{5} \right) - V_{a-a} = 0 \quad V_{a-a} = P$$

The cross-sectional area of section *a-a* is $A_{a-a} = (0.025) \left(\frac{0.025}{3/5} \right) = 1.0417(10^{-3}) \text{ m}^2$. For Normal stress,

$$\sigma_{\text{allow}} = \frac{N_{a-a}}{A_{a-a}}; \quad 150(10^6) = \frac{0.75P}{1.0417(10^{-3})}$$

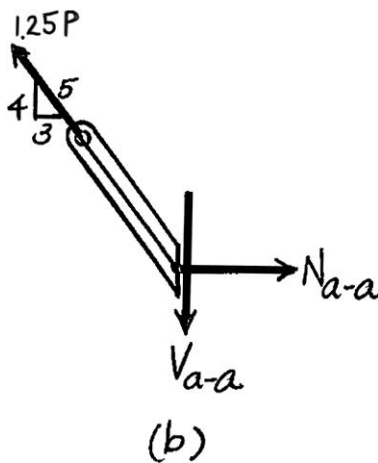
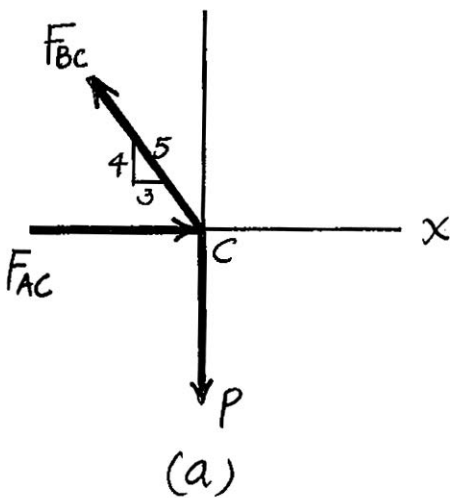
$$P = 208.33(10^3) \text{ N} = 208.33 \text{ kN}$$

For Shear Stress

$$\tau_{\text{allow}} = \frac{V_{a-a}}{A_{a-a}}; \quad 60(10^6) = \frac{P}{1.0417(10^{-3})}$$

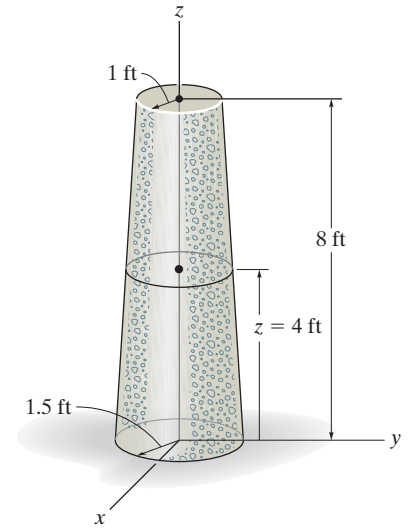
$$P = 62.5(10^3) \text{ N} = 62.5 \text{ kN (Controls!)}$$

Ans.



Ans:
 $P = 62.5 \text{ kN}$

1-67. The pedestal in the shape of a frustum of a cone is made of concrete having a specific weight of 150 lb/ft^3 . Determine the average normal stress acting in the pedestal at its base. *Hint:* The volume of a cone of radius r and height h is $V = \frac{1}{3}\pi r^2 h$.



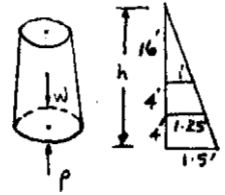
$$\frac{h}{1.5} = \frac{h - 8}{1}, \quad h = 24 \text{ ft}$$

$$V = \frac{1}{3}\pi (1.5)^2(24) - \frac{1}{3}\pi (1)^2(16); \quad V = 39.794 \text{ ft}^3$$

$$W = 150(39.794) = 5.969 \text{ kip}$$

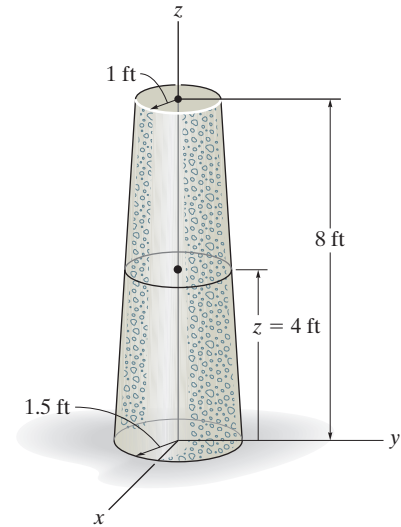
$$\sigma = \frac{P}{A} = \frac{5.969}{\pi(1.5)^2} = 844 \text{ psf} = 5.86 \text{ psi}$$

Ans.



Ans:
 $\sigma = 5.86 \text{ psi}$

***1-68.** The pedestal in the shape of a frustum of a cone is made of concrete having a specific weight of 150 lb/ft^3 . Determine the average normal stress acting in the pedestal at its midheight, $z = 4 \text{ ft}$. *Hint:* The volume of a cone of radius r and height h is $V = \frac{1}{3}\pi r^2 h$.



$$\frac{h}{1.5} = \frac{h - 8}{1}, \quad h = 24 \text{ ft}$$

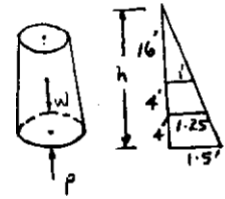
$$W = \left[\frac{1}{3} \pi (1.25)^2 20 - \frac{1}{3} (\pi) (1^2) (16) \right] (150) = 2395.5 \text{ lb}$$

$$+\uparrow \Sigma F_y = 0; \quad P - 2395.5 = 0$$

$$P = 2395.5 \text{ lb}$$

$$\sigma = \frac{P}{A} = \frac{2395.5}{\pi (1.25)^2} = 488 \text{ psf} = 3.39 \text{ psi}$$

Ans.



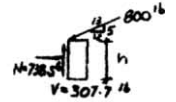
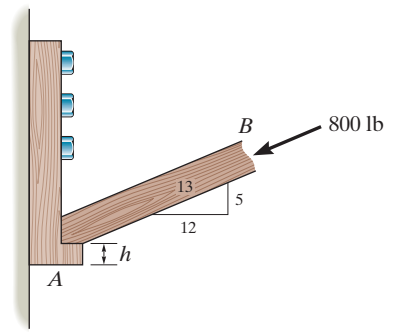
1-69. Member B is subjected to a compressive force of 800 lb. If A and B are both made of wood and are $\frac{3}{8}$ in. thick, determine to the nearest $\frac{1}{4}$ in. the smallest dimension h of the horizontal segment so that it does not fail in shear. The average shear stress for the segment is $\tau_{\text{allow}} = 300$ psi.

$$\tau_{\text{allow}} = 300 = \frac{307.7}{\left(\frac{3}{8}\right)h}$$

$$h = 2.74 \text{ in.}$$

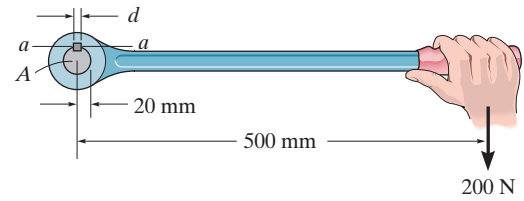
$$\text{Use } h = 2\frac{3}{4} \text{ in.}$$

Ans.



Ans:
Use $h = 2\frac{3}{4}$ in.

1-70. The lever is attached to the shaft A using a key that has a width d and length of 25 mm. If the shaft is fixed and a vertical force of 200 N is applied perpendicular to the handle, determine the dimension d if the allowable shear stress for the key is $\tau_{\text{allow}} = 35 \text{ MPa}$.

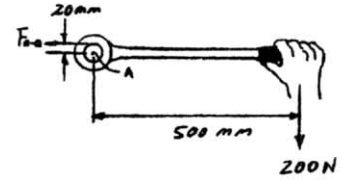


$$\zeta + \Sigma M_A = 0; \quad F_{a-a}(20) - 200(500) = 0$$

$$F_{a-a} = 5000 \text{ N}$$

$$\tau_{\text{allow}} = \frac{F_{a-a}}{A_{a-a}}; \quad 35(10^6) = \frac{5000}{d(0.025)}$$

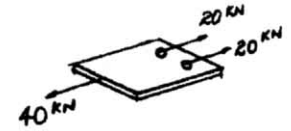
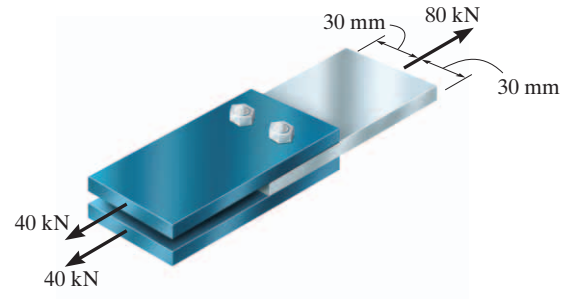
$$d = 0.00571 \text{ m} = 5.71 \text{ mm}$$



Ans.

Ans:
 $d = 5.71 \text{ mm}$

1-71. The joint is fastened together using two bolts. Determine the required diameter of the bolts if the failure shear stress for the bolts is $\tau_{\text{fail}} = 350 \text{ MPa}$. Use a factor of safety for shear of F.S. = 2.5.



$$\frac{350(10^6)}{2.5} = 140(10^6)$$

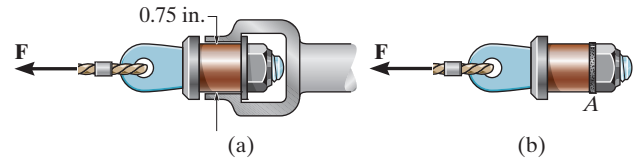
$$\tau_{\text{allow}} = 140(10^6) = \frac{20(10^3)}{\frac{\pi}{4} d^2}$$

$$d = 0.0135 \text{ m} = 13.5 \text{ mm}$$

Ans.

Ans:
 $d = 13.5 \text{ mm}$

1-73. The steel swivel bushing in the elevator control of an airplane is held in place using a nut and washer as shown in Fig. (a). Failure of the washer *A* can cause the push rod to separate as shown in Fig. (b). If the maximum average normal stress for the washer is $\sigma_{\max} = 60$ ksi and the maximum average shear stress is $\tau_{\max} = 21$ ksi, determine the force **F** that must be applied to the bushing that will cause this to happen. The washer is $\frac{1}{16}$ in. thick.



$$\tau_{\text{avg}} = \frac{V}{A};$$

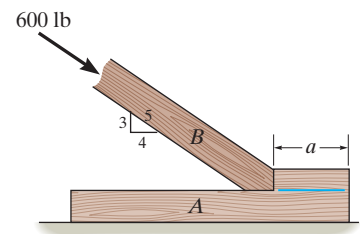
$$21(10^3) = \frac{F}{2\pi(0.375)(\frac{1}{16})}$$

$$F = 3092.5 \text{ lb} = 3.09 \text{ kip}$$

Ans.

Ans:
 $F = 3.09 \text{ kip}$

1-74. Member B is subjected to a compressive force of 600 lb. If A and B are both made of wood and are 1.5 in. thick, determine to the nearest $\frac{1}{8}$ in. the smallest dimension a of the support so that the average shear stress along the blue line does not exceed $\tau_{\text{allow}} = 50$ psi. Neglect friction.



Consider the equilibrium of the FBD of member B , Fig. a ,

$$\rightarrow \Sigma F_x = 0; \quad 600\left(\frac{4}{5}\right) - F_h = 0 \quad F_h = 480 \text{ lb}$$

Referring to the FBD of the wood segment sectioned through glue line, Fig. b

$$\rightarrow \Sigma F_x = 0; \quad 480 - V = 0 \quad V = 480 \text{ lb}$$

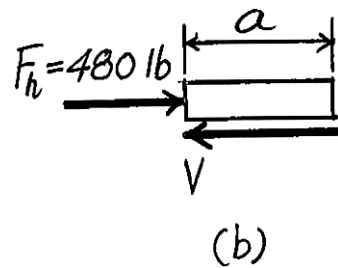
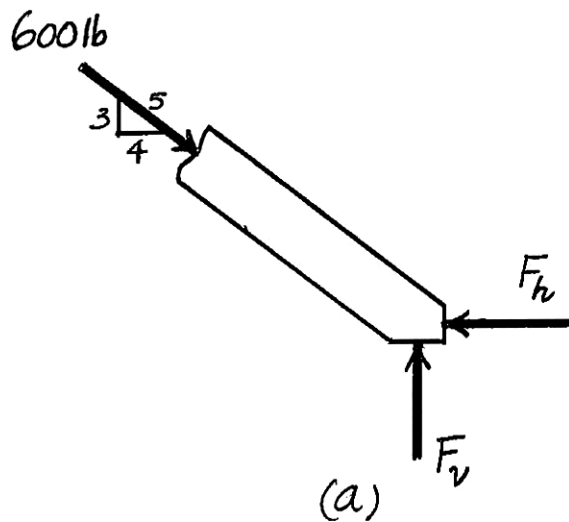
The area of shear plane is $A = 1.5(a)$. Thus,

$$\tau_{\text{allow}} = \frac{V}{A}; \quad 50 = \frac{480}{1.5a}$$

$$a = 6.40 \text{ in.}$$

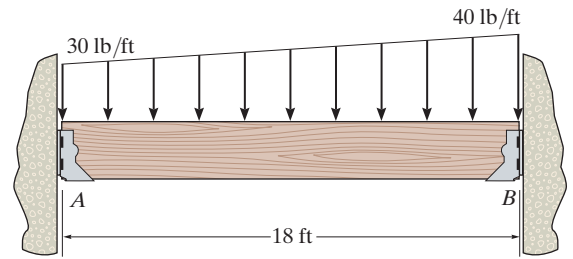
$$\text{Use } a = 6\frac{1}{2} \text{ in.}$$

Ans.



Ans:
Use $a = 6\frac{1}{2}$ in.

1-75. The hangers support the joist uniformly, so that it is assumed the four nails on each hanger carry an equal portion of the load. If the joist is subjected to the loading shown, determine the average shear stress in each nail of the hanger at ends *A* and *B*. Each nail has a diameter of 0.25 in. The hangers only support vertical loads.



$$\zeta + \Sigma M_A = 0; \quad F_B(18) - 540(9) - 90(12) = 0; \quad F_B = 330 \text{ lb}$$

$$+\uparrow \Sigma F_y = 0; \quad F_A + 330 - 540 - 90 = 0; \quad F_A = 300 \text{ lb}$$

For nails at *A*,

$$\tau_{\text{avg}} = \frac{F_A}{A_A} = \frac{300}{4\left(\frac{\pi}{4}\right)(0.25)^2}$$

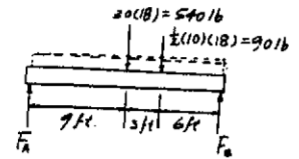
$$= 1528 \text{ psi} = 1.53 \text{ ksi}$$

For nails at *B*,

$$\tau_{\text{avg}} = \frac{F_B}{A_B} = \frac{330}{4\left(\frac{\pi}{4}\right)(0.25)^2}$$

$$= 1681 \text{ psi} = 1.68 \text{ ksi}$$

Ans.



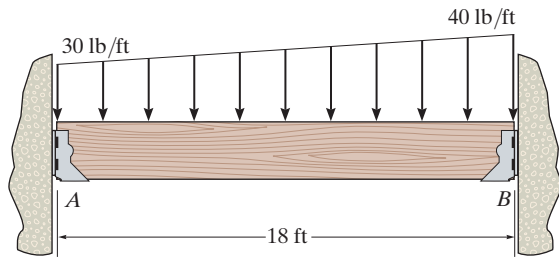
Ans.

Ans:

For nails at *A*: $\tau_{\text{avg}} = 1.53 \text{ ksi}$

For nails at *B*: $\tau_{\text{avg}} = 1.68 \text{ ksi}$

***1-76.** The hangers support the joists uniformly, so that it is assumed the four nails on each hanger carry an equal portion of the load. Determine the smallest diameter of the nails at A and at B if the allowable stress for the nails is $\tau_{\text{allow}} = 4 \text{ ksi}$. The hangers only support vertical loads.



$$\zeta + \Sigma M_A = 0; \quad F_B(18) - 540(9) - 90(12) = 0; \quad F_B = 330 \text{ lb}$$

$$+\uparrow \Sigma F_y = 0; \quad F_A + 330 - 540 - 90 = 0; \quad F_A = 300 \text{ lb}$$

For nails at A ,

$$\tau_{\text{allow}} = \frac{F_A}{A_A}; \quad 4(10^3) = \frac{300}{4\left(\frac{\pi}{4}\right)d_A^2}$$

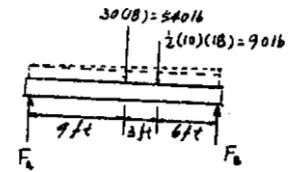
$$d_A = 0.155 \text{ in.}$$

For nails at B ,

$$\tau_{\text{allow}} = \frac{F_B}{A_B}; \quad 4(10^3) = \frac{330}{4\left(\frac{\pi}{4}\right)d_B^2}$$

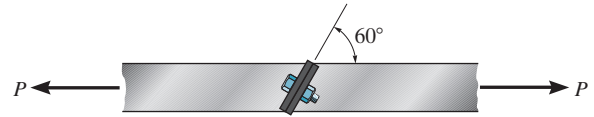
$$d_B = 0.162 \text{ in.}$$

Ans.



Ans.

1-77. The tension member is fastened together using *two* bolts, one on each side of the member as shown. Each bolt has a diameter of 0.3 in. Determine the maximum load P that can be applied to the member if the allowable shear stress for the bolts is $\tau_{\text{allow}} = 12$ ksi and the allowable average normal stress is $\sigma_{\text{allow}} = 20$ ksi.



$$\uparrow + \Sigma F_y = 0; \quad N - P \sin 60^\circ = 0$$

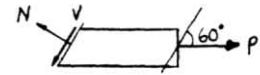
$$P = 1.1547 N$$

(1)

$$\leftarrow + \Sigma F_x = 0; \quad V - P \cos 60^\circ = 0$$

$$P = 2V$$

(2)



Assume failure due to shear:

$$\tau_{\text{allow}} = 12 = \frac{V}{(2) \frac{\pi}{4} (0.3)^2}$$

$$V = 1.696 \text{ kip}$$

From Eq. (2),

$$P = 3.39 \text{ kip}$$

Assume failure due to normal force:

$$\sigma_{\text{allow}} = 20 = \frac{N}{(2) \frac{\pi}{4} (0.3)^2}$$

$$N = 2.827 \text{ kip}$$

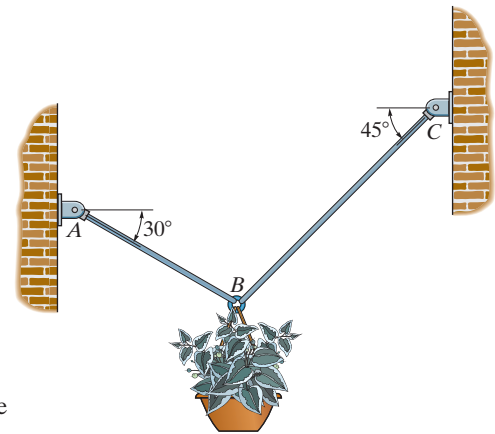
From Eq. (1),

$$P = 3.26 \text{ kip} \quad (\text{controls})$$

Ans.

Ans:
 $P = 3.26 \text{ kip}$

1-78. The 50-kg flowerpot is suspended from wires AB and BC . If the wires have a normal failure stress of $\sigma_{fail} = 350$ MPa, determine the minimum diameter of each wire. Use a factor of safety of 2.5.



Internal Loading: The normal force developed in cables AB and BC can be determined by considering the equilibrium of joint B , Fig. a .

$$\rightarrow \Sigma F_x = 0; \quad F_{BC} \cos 45^\circ - F_{AB} \cos 30^\circ = 0 \quad (1)$$

$$+\uparrow \Sigma F_y = 0; \quad F_{AB} \sin 30^\circ + F_{BC} \sin 45^\circ - 50(9.81) = 0 \quad (2)$$

Solving Eqs. (1) and (2),

$$F_{AB} = 359.07 \text{ N} \quad F_{BC} = 439.77 \text{ N}$$

Allowable Normal Stress:

$$\sigma_{allow} = \frac{\sigma_{fail}}{\text{F.S.}} = \frac{350}{2.5} = 140 \text{ MPa}$$

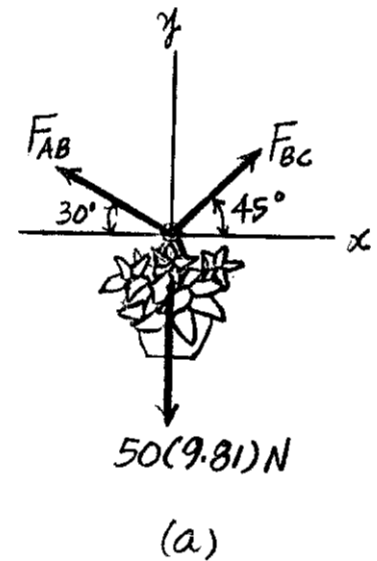
Using this result,

$$\sigma_{allow} = \frac{F_{AB}}{A_{AB}}; \quad 140(10^6) = \frac{359.07}{\frac{\pi}{4} d_{AB}^2}$$

$$d_{AB} = 0.001807 \text{ m} = 1.81 \text{ mm} \quad \text{Ans.}$$

$$\sigma_{allow} = \frac{F_{BC}}{A_{BC}}; \quad 140(10^6) = \frac{439.77}{\frac{\pi}{4} d_{BC}^2}$$

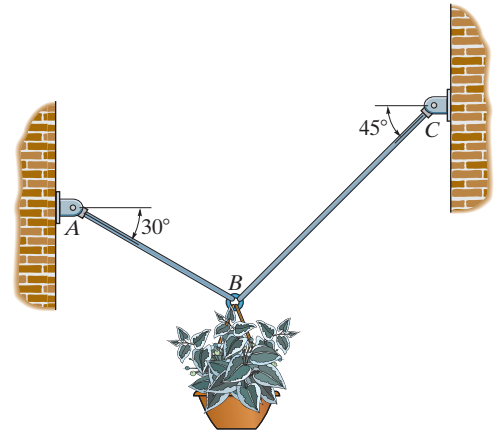
$$d_{BC} = 0.00200 \text{ m} = 2.00 \text{ mm} \quad \text{Ans.}$$



Ans:

$$d_{AB} = 1.81 \text{ mm}, d_{BC} = 2.00 \text{ mm}$$

1-79. The 50-kg flowerpot is suspended from wires AB and BC which have diameters of 1.5 mm and 2 mm, respectively. If the wires have a normal failure stress of $\sigma_{\text{fail}} = 350$ MPa, determine the factor of safety of each wire.



Internal Loading: The normal force developed in cables AB and BC can be determined by considering the equilibrium of joint B , Fig. a .

$$\rightarrow \Sigma F_x = 0; \quad F_{BC} \cos 45^\circ - F_{AB} \cos 30^\circ = 0 \quad (1)$$

$$+\uparrow \Sigma F_y = 0; \quad F_{AB} \sin 30^\circ + F_{BC} \sin 45^\circ - 50(9.81) = 0 \quad (2)$$

Solving Eqs. (1) and (2),

$$F_{AB} = 359.07 \text{ N} \quad F_{BC} = 439.77 \text{ N}$$

Average Normal Stress: The cross-sectional area of wires AB and BC are

$$A_{AB} = \frac{\pi}{4} (0.0015)^2 = 1.767(10^{-6}) \text{ m}^2 \text{ and } A_{BC} = \frac{\pi}{4} (0.002)^2 = 3.142(10^{-6}) \text{ m}^2.$$

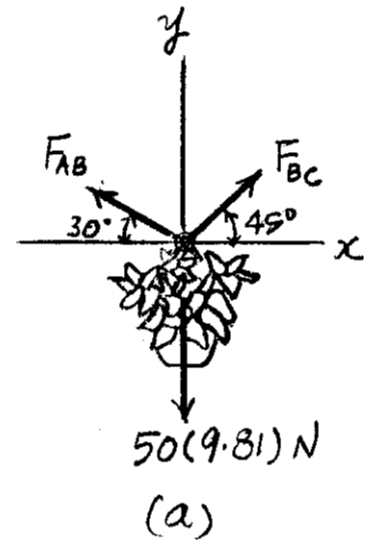
$$(\sigma_{\text{avg}})_{AB} = \frac{F_{AB}}{A_{AB}} = \frac{359.07}{1.767(10^{-6})} = 203.19 \text{ MPa}$$

$$(\sigma_{\text{avg}})_{BC} = \frac{F_{BC}}{A_{BC}} = \frac{439.77}{3.142(10^{-6})} = 139.98 \text{ MPa}$$

We obtain,

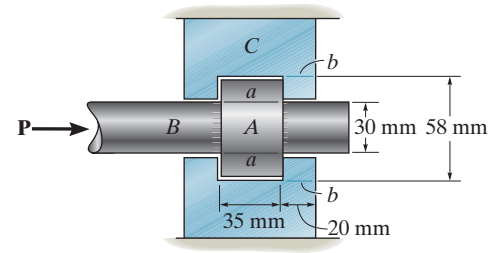
$$(\text{F.S.})_{AB} = \frac{\sigma_{\text{fail}}}{(\sigma_{\text{avg}})_{AB}} = \frac{350}{203.19} = 1.72 \quad \text{Ans.}$$

$$(\text{F.S.})_{BC} = \frac{\sigma_{\text{fail}}}{(\sigma_{\text{avg}})_{BC}} = \frac{350}{139.98} = 2.50 \quad \text{Ans.}$$



Ans:
 $(\text{F.S.})_{AB} = 1.72, (\text{F.S.})_{BC} = 2.50$

***1-80.** The thrust bearing consists of a circular collar A fixed to the shaft B . Determine the maximum axial force P that can be applied to the shaft so that it does not cause the shear stress along a cylindrical surface a or b to exceed an allowable shear stress of $\tau_{\text{allow}} = 170 \text{ MPa}$.



Assume failure along a :

$$\tau_{\text{allow}} = 170(10^6) = \frac{P}{\pi(0.03)(0.035)}$$

$$P = 561 \text{ kN (controls)}$$

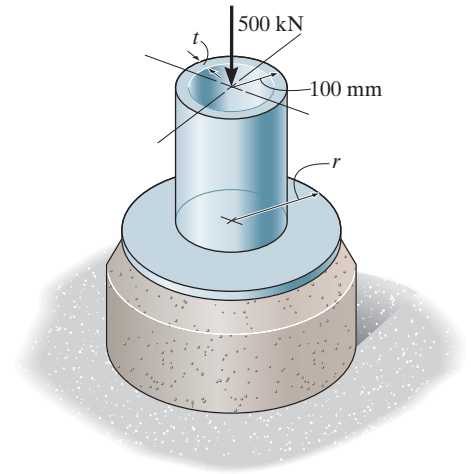
Ans.

Assume failure along b :

$$\tau_{\text{allow}} = 170(10^6) = \frac{P}{\pi(0.058)(0.02)}$$

$$P = 620 \text{ kN}$$

1–81. The steel pipe is supported on the circular base plate and concrete pedestal. If the normal failure stress for the steel is $(\sigma_{\text{fail}})_{\text{st}} = 350$ MPa, determine the minimum thickness t of the pipe if it supports the force of 500 kN. Use a factor of safety against failure of 1.5. Also, find the minimum radius r of the base plate so that the minimum factor of safety against failure of the concrete due to bearing is 2.5. The failure bearing stress for concrete is $(\sigma_{\text{fail}})_{\text{con}} = 25$ MPa.



Allowable Stress:

$$(\sigma_{\text{allow}})_{\text{st}} = \frac{(\sigma_{\text{fail}})_{\text{st}}}{\text{F.S.}} = \frac{350}{1.5} = 233.33 \text{ MPa}$$

$$(\sigma_{\text{allow}})_{\text{con}} = \frac{(\sigma_{\text{fail}})_{\text{con}}}{\text{F.S.}} = \frac{25}{2.5} = 10 \text{ MPa}$$

The cross-sectional area of the steel pipe and the bearing area of the concrete pedestal are $A_{\text{st}} = \pi(0.1^2 - r_i^2)$ and $(A_{\text{con}})_{\text{b}} = \pi r^2$. Using these results,

$$(\sigma_{\text{allow}})_{\text{st}} = \frac{P}{A_{\text{st}}}; \quad 233.33(10^6) = \frac{500(10^3)}{\pi(0.1^2 - r_i^2)}$$

$$r_i = 0.09653 \text{ m} = 96.53 \text{ mm}$$

Thus, the minimum required thickness of the steel pipe is

$$t = r_O - r_i = 100 - 96.53 = 3.47 \text{ mm} \quad \text{Ans.}$$

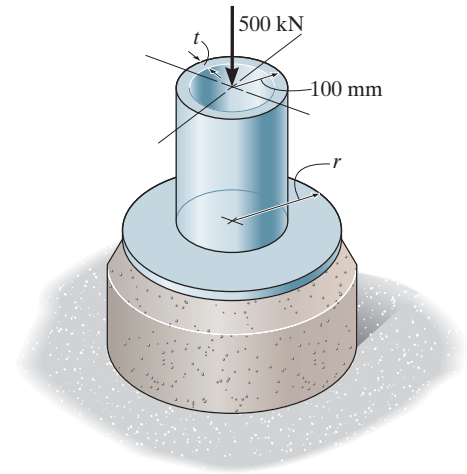
The minimum required radius of the bearing area of the concrete pedestal is

$$(\sigma_{\text{allow}})_{\text{con}} = \frac{P}{(A_{\text{con}})_{\text{b}}}; \quad 10(10^6) = \frac{500(10^3)}{\pi r^2}$$

$$r = 0.1262 \text{ m} = 126 \text{ mm} \quad \text{Ans.}$$

Ans:
 $t = 3.47 \text{ mm}, r = 126 \text{ mm}$

1-82. The steel pipe is supported on the circular base plate and concrete pedestal. If the thickness of the pipe is $t = 5$ mm and the base plate has a radius of 150 mm, determine the factors of safety against failure of the steel and concrete. The applied force is 500 kN, and the normal failure stresses for steel and concrete are $(\sigma_{fail})_{st} = 350$ MPa and $(\sigma_{fail})_{con} = 25$ MPa, respectively.



Average Normal and Bearing Stress: The cross-sectional area of the steel pipe and the bearing area of the concrete pedestal are $A_{st} = \pi(0.1^2 - 0.095^2) = 0.975(10^{-3})\pi$ m² and $(A_{con})_b = \pi(0.15^2) = 0.0225\pi$ m². We have

$$(\sigma_{avg})_{st} = \frac{P}{A_{st}} = \frac{500(10^3)}{0.975(10^{-3})\pi} = 163.24 \text{ MPa}$$

$$(\sigma_{avg})_{con} = \frac{P}{(A_{con})_b} = \frac{500(10^3)}{0.0225\pi} = 7.074 \text{ MPa}$$

Thus, the factor of safety against failure of the steel pipe and concrete pedestal are

$$(\text{F.S.})_{st} = \frac{(\sigma_{fail})_{st}}{(\sigma_{avg})_{st}} = \frac{350}{163.24} = 2.14 \quad \text{Ans.}$$

$$(\text{F.S.})_{con} = \frac{(\sigma_{fail})_{con}}{(\sigma_{avg})_{con}} = \frac{25}{7.074} = 3.53 \quad \text{Ans.}$$

Ans:

$$(\text{F.S.})_{st} = 2.14, (\text{F.S.})_{con} = 3.53$$

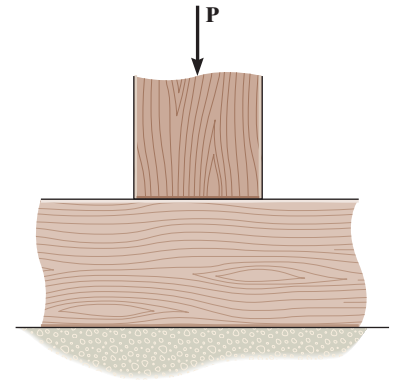
1-83. The 60 mm × 60 mm oak post is supported on the pine block. If the allowable bearing stresses for these materials are $\sigma_{\text{oak}} = 43 \text{ MPa}$ and $\sigma_{\text{pine}} = 25 \text{ MPa}$, determine the greatest load P that can be supported. If a rigid bearing plate is used between these materials, determine its required area so that the maximum load P can be supported. What is this load?

For failure of pine block:

$$\sigma = \frac{P}{A}; \quad 25(10^6) = \frac{P}{(0.06)(0.06)}$$

$$P = 90 \text{ kN}$$

Ans.



For failure of oak post:

$$\sigma = \frac{P}{A}; \quad 43(10^6) = \frac{P}{(0.06)(0.06)}$$

$$P = 154.8 \text{ kN}$$

Area of plate based on strength of pine block:

$$\sigma = \frac{P}{A}; \quad 25(10^6) = \frac{154.8(10^3)}{A}$$

$$A = 6.19(10^{-3}) \text{ m}^2$$

$$P_{\text{max}} = 155 \text{ kN}$$

Ans.

Ans.

Ans:

$$P = 90 \text{ kN}, A = 6.19(10^{-3}) \text{ m}^2, P_{\text{max}} = 155 \text{ kN}$$

*1-84. The frame is subjected to the load of 4 kN which acts on member ABD at D . Determine the required diameter of the pins at D and C if the allowable shear stress for the material is $\tau_{\text{allow}} = 40 \text{ MPa}$. Pin C is subjected to double shear, whereas pin D is subjected to single shear.

Referring to the FBD of member DCE , Fig. a ,

$$\zeta + \Sigma M_E = 0; \quad D_y(2.5) - F_{BC} \sin 45^\circ (1) = 0 \quad (1)$$

$$\rightarrow \Sigma F_x = 0 \quad F_{BC} \cos 45^\circ - D_x = 0 \quad (2)$$

Referring to the FBD of member ABD , Fig. b ,

$$\zeta + \Sigma M_A = 0; \quad 4 \cos 45^\circ (3) + F_{BC} \sin 45^\circ (1.5) - D_x (3) = 0 \quad (3)$$

Solving Eqs (2) and (3),

$$F_{BC} = 8.00 \text{ kN} \quad D_x = 5.657 \text{ kN}$$

Substitute the result of F_{BC} into (1)

$$D_y = 2.263 \text{ kN}$$

Thus, the force acting on pin D is

$$F_D = \sqrt{D_x^2 + D_y^2} = \sqrt{5.657^2 + 2.263^2} = 6.093 \text{ kN}$$

Pin C is subjected to double shear while pin D is subjected to single shear. Referring to the FBDs of pins C , and D in Fig c and d , respectively,

$$V_C = \frac{F_{BC}}{2} = \frac{8.00}{2} = 4.00 \text{ kN} \quad V_D = F_D = 6.093 \text{ kN}$$

For pin C ,

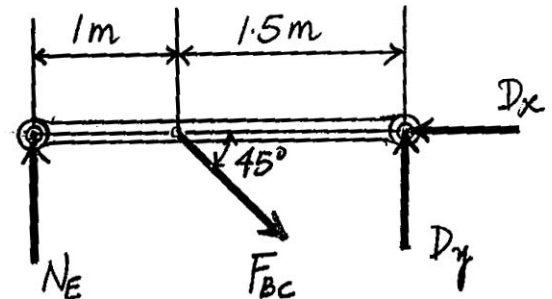
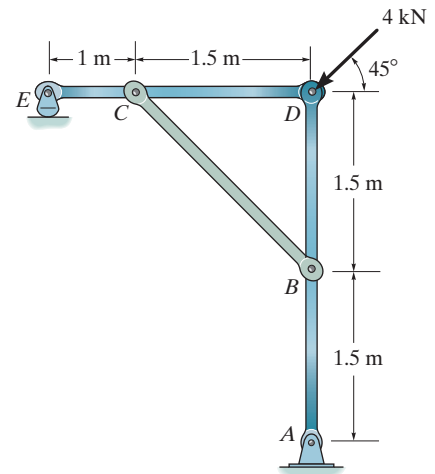
$$\tau_{\text{allow}} = \frac{V_C}{A_C}; \quad 40(10^6) = \frac{4.00(10^3)}{\frac{\pi}{4} d_C^2}$$

$$d_C = 0.01128 \text{ m} = 11.3 \text{ mm}$$

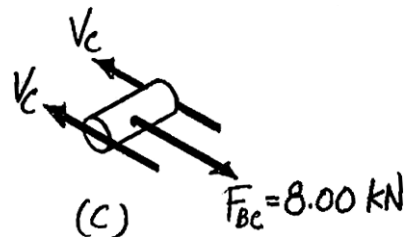
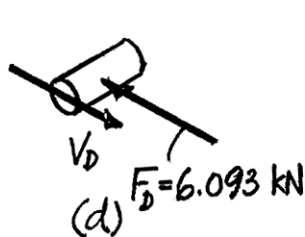
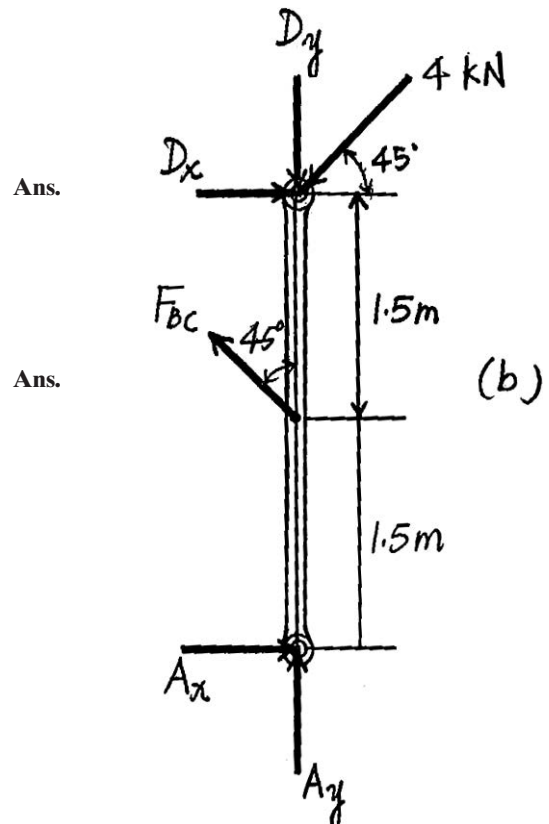
For pin D ,

$$\tau_{\text{allow}} = \frac{V_D}{A_D}; \quad 40(10^6) = \frac{6.093(10^3)}{\frac{\pi}{4} d_D^2}$$

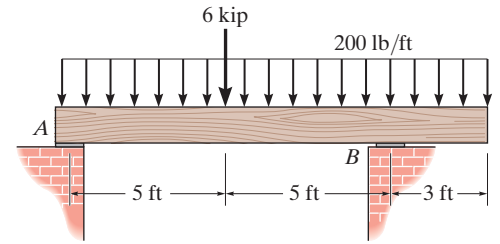
$$d_D = 0.01393 \text{ m} = 13.9 \text{ mm}$$



(a)



1-85. The beam is made from southern pine and is supported by base plates resting on brick work. If the allowable bearing stresses for the materials are $(\sigma_{\text{pine}})_{\text{allow}} = 2.81 \text{ ksi}$ and $(\sigma_{\text{brick}})_{\text{allow}} = 6.70 \text{ ksi}$, determine the required length of the base plates at A and B to the nearest $\frac{1}{4}$ inch in order to support the load shown. The plates are 3 in. wide.



The design must be based on strength of the pine.

At A :

$$\sigma = \frac{P}{A}; \quad 2810 = \frac{3910}{l_A(3)}$$

$$\text{Use } l_A = \frac{1}{2} \text{ in.} \quad l_A = 0.464 \text{ in.}$$

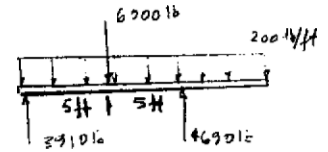
Ans.

At B :

$$\sigma = \frac{P}{A}; \quad 2810 = \frac{4690}{l_B(3)}$$

$$\text{Use } l_B = \frac{3}{4} \text{ in.} \quad l_B = 0.556 \text{ in.}$$

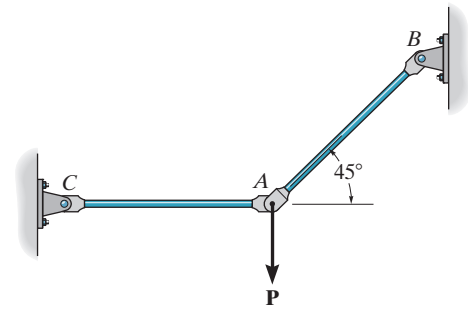
Ans.



Ans:

$$\text{Use } l_A = \frac{1}{2} \text{ in.}, l_B = \frac{3}{4} \text{ in.}$$

1-86. The two aluminum rods support the vertical force of $P = 20$ kN. Determine their required diameters if the allowable tensile stress for the aluminum is $\sigma_{\text{allow}} = 150$ MPa.



$$+\uparrow \Sigma F_y = 0; \quad F_{AB} \sin 45^\circ - 20 = 0; \quad F_{AB} = 28.284 \text{ kN}$$

$$\rightarrow \Sigma F_x = 0; \quad 28.284 \cos 45^\circ - F_{AC} = 0; \quad F_{AC} = 20.0 \text{ kN}$$

For rod AB:

$$\sigma_{\text{allow}} = \frac{F_{AB}}{A_{AB}}; \quad 150(10^6) = \frac{28.284(10^3)}{\frac{\pi}{4} d_{AB}^2}$$

$$d_{AB} = 0.0155 \text{ m} = 15.5 \text{ mm}$$

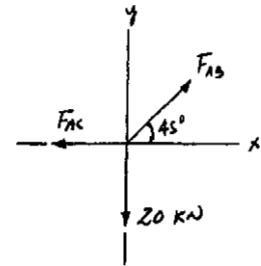
Ans.

For rod AC:

$$\sigma_{\text{allow}} = \frac{F_{AC}}{A_{AC}}; \quad 150(10^6) = \frac{20.0(10^3)}{\frac{\pi}{4} d_{AC}^2}$$

$$d_{AC} = 0.0130 \text{ m} = 13.0 \text{ mm}$$

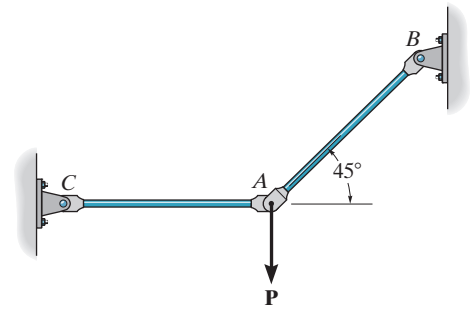
Ans.



Ans:

$$d_{AB} = 15.5 \text{ mm}, \quad d_{AC} = 13.0 \text{ mm}$$

1-87. The two aluminum rods AB and AC have diameters of 10 mm and 8 mm, respectively. Determine the largest vertical force P that can be supported. The allowable tensile stress for the aluminum is $\sigma_{\text{allow}} = 150 \text{ MPa}$.



$$+\uparrow \Sigma F_y = 0; \quad F_{AB} \sin 45^\circ - P = 0; \quad P = F_{AB} \sin 45^\circ \quad (1)$$

$$\rightarrow \Sigma F_x = 0; \quad F_{AB} \cos 45^\circ - F_{AC} = 0 \quad (2)$$

Assume failure of rod AB :

$$\sigma_{\text{allow}} = \frac{F_{AB}}{A_{AB}}; \quad 150(10^6) = \frac{F_{AB}}{\frac{\pi}{4}(0.01)^2}$$

$$F_{AB} = 11.78 \text{ kN}$$

From Eq. (1),

$$P = 8.33 \text{ kN}$$

Assume failure of rod AC :

$$\sigma_{\text{allow}} = \frac{F_{AC}}{A_{AC}}; \quad 150(10^6) = \frac{F_{AC}}{\frac{\pi}{4}(0.008)^2}$$

$$F_{AC} = 7.540 \text{ kN}$$

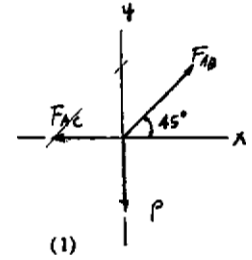
Solving Eqs. (1) and (2) yields:

$$F_{AB} = 10.66 \text{ kN}; \quad P = 7.54 \text{ kN}$$

Choose the smallest value

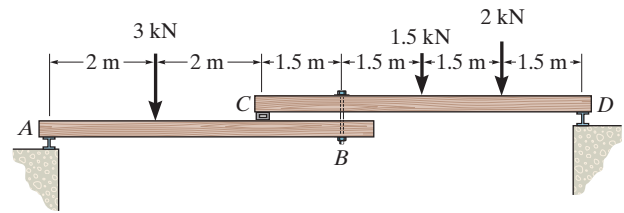
$$P = 7.54 \text{ kN}$$

Ans.



Ans:
 $P = 7.54 \text{ kN}$

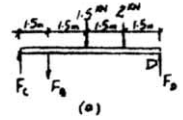
***1-88.** The compound wooden beam is connected together by a bolt at B . Assuming that the connections at A , B , C , and D exert only vertical forces on the beam, determine the required diameter of the bolt at B and the required outer diameter of its washers if the allowable tensile stress for the bolt is $(\sigma_t)_{\text{allow}} = 150 \text{ MPa}$ and the allowable bearing stress for the wood is $(\sigma_b)_{\text{allow}} = 28 \text{ MPa}$. Assume that the hole in the washers has the same diameter as the bolt.



From FBD (a):

$$\zeta + \sum M_D = 0; \quad F_B(4.5) + 1.5(3) + 2(1.5) - F_C(6) = 0$$

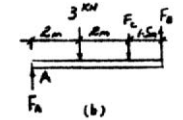
$$4.5 F_B - 6 F_C = -7.5 \quad (1)$$



From FBD (b):

$$\zeta + \sum M_A = 0; \quad F_B(5.5) - F_C(4) - 3(2) = 0$$

$$5.5 F_B - 4 F_C = 6 \quad (2)$$



Solving Eqs. (1) and (2) yields

$$F_B = 4.40 \text{ kN}; \quad F_C = 4.55 \text{ kN}$$

For bolt:

$$\sigma_{\text{allow}} = 150(10^6) = \frac{4.40(10^3)}{\frac{\pi}{4}(d_B)^2}$$

$$d_B = 0.00611 \text{ m}$$

$$= 6.11 \text{ mm}$$

Ans.

For washer:

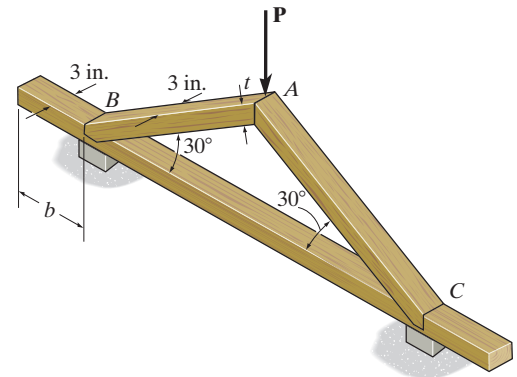
$$\sigma_{\text{allow}} = 28(10^4) = \frac{4.40(10^3)}{\frac{\pi}{4}(d_w^2 - 0.00611^2)}$$

$$d_w = 0.0154 \text{ m} = 15.4 \text{ mm}$$

Ans.



1-89. Determine the required minimum thickness t of member AB and edge distance b of the frame if $P = 9$ kip and the factor of safety against failure is 2. The wood has a normal failure stress of $\sigma_{fail} = 6$ ksi, and shear failure stress of $\tau_{fail} = 1.5$ ksi.



Internal Loadings: The normal force developed in member AB can be determined by considering the equilibrium of joint A . Fig. a .

$$\begin{aligned} \rightarrow \Sigma F_x = 0; & \quad F_{AB} \cos 30^\circ - F_{AC} \cos 30^\circ = 0 & \quad F_{AC} = F_{AB} \\ + \uparrow \Sigma F_y = 0; & \quad 2F_{AB} \sin 30^\circ - 9 = 0 & \quad F_{AB} = 9 \text{ kip} \end{aligned}$$

Subsequently, the horizontal component of the force acting on joint B can be determined by analyzing the equilibrium of member AB , Fig. b .

$$\rightarrow \Sigma F_x = 0; \quad (F_B)_x - 9 \cos 30^\circ = 0 \quad (F_B)_x = 7.794 \text{ kip}$$

Referring to the free-body diagram shown in Fig. c , the shear force developed on the shear plane $a-a$ is

$$\rightarrow \Sigma F_x = 0; \quad V_{a-a} - 7.794 = 0 \quad V_{a-a} = 7.794 \text{ kip}$$

Allowable Normal Stress:

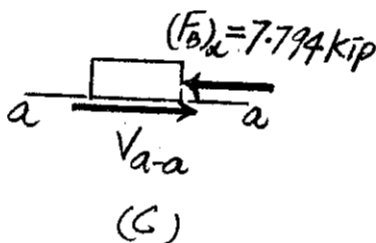
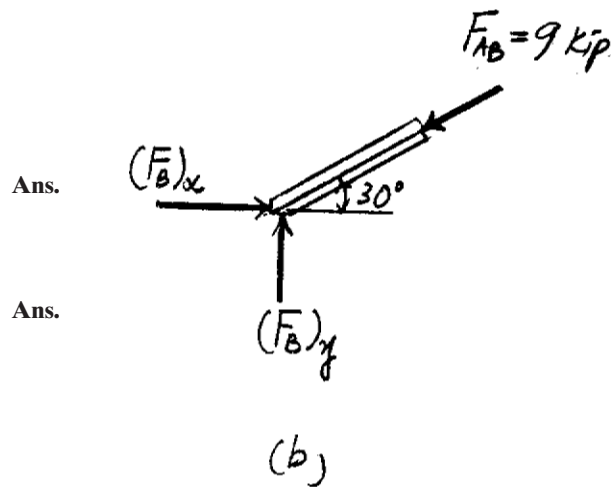
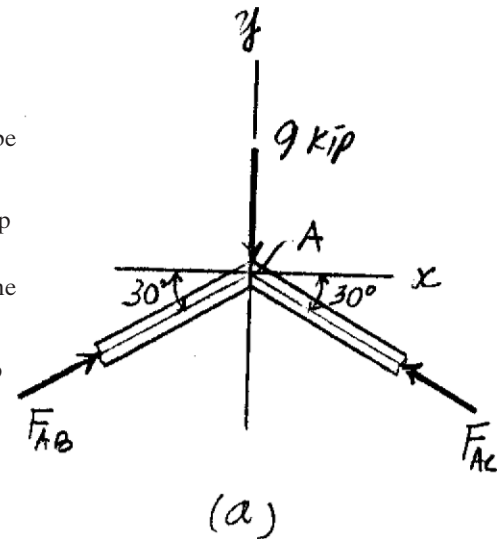
$$\sigma_{allow} = \frac{\sigma_{fail}}{\text{F.S.}} = \frac{6}{2} = 3 \text{ ksi}$$

$$\tau_{allow} = \frac{\tau_{fail}}{\text{F.S.}} = \frac{1.5}{2} = 0.75 \text{ ksi}$$

Using these results,

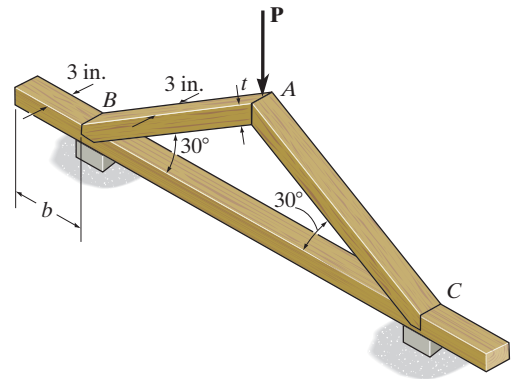
$$\sigma_{allow} = \frac{F_{AB}}{A_{AB}}; \quad 3(10^3) = \frac{9(10^3)}{3t} \quad t = 1 \text{ in.}$$

$$\tau_{allow} = \frac{V_{a-a}}{A_{a-a}}; \quad 0.75(10^3) = \frac{7.794(10^3)}{3b} \quad b = 3.46 \text{ in.}$$



Ans:
 $t = 1 \text{ in.}, b = 3.46 \text{ in.}$

1-90. Determine the maximum allowable load P that can be safely supported by the frame if $t = 1.25$ in. and $b = 3.5$ in. The wood has a normal failure stress of $\sigma_{fail} = 6$ ksi, and shear failure stress of $\tau_{fail} = 1.5$ ksi. Use a factor of safety against failure of 2.



Internal Loadings: The normal force developed in member AB can be determined by considering the equilibrium of joint A , Fig. a .

$$\rightarrow \Sigma F_x = 0; \quad F_{AB} \cos 30^\circ - F_{AC} \cos 30^\circ = 0 \quad F_{AC} = F_{AB}$$

$$+\uparrow \Sigma F_y = 0; \quad 2F_{AB} \sin 30^\circ - 9 = 0 \quad F_{AB} = P$$

Subsequently, the horizontal component of the force acting on joint B can be determined by analyzing the equilibrium of member AB , Fig. b .

$$\rightarrow \Sigma F_x = 0; \quad (F_B)_x - P \cos 30^\circ = 0 \quad (F_B)_x = 0.8660P$$

Referring to the free-body diagram shown in Fig. c , the shear force developed on the shear plane $a-a$ is

$$\rightarrow \Sigma F_x = 0; \quad V_{a-a} - 0.8660P = 0 \quad V_{a-a} = 0.8660P$$

Allowable Normal and Shear Stress:

$$\sigma_{allow} = \frac{\sigma_{fail}}{F.S.} = \frac{6}{2} = 3 \text{ ksi}$$

$$\tau_{allow} = \frac{\tau_{fail}}{F.S.} = \frac{1.5}{2} = 0.75 \text{ ksi}$$

Using these results,

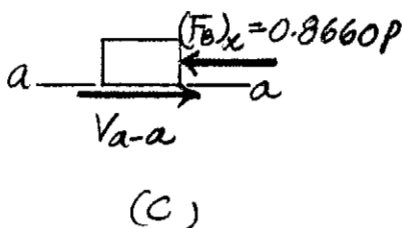
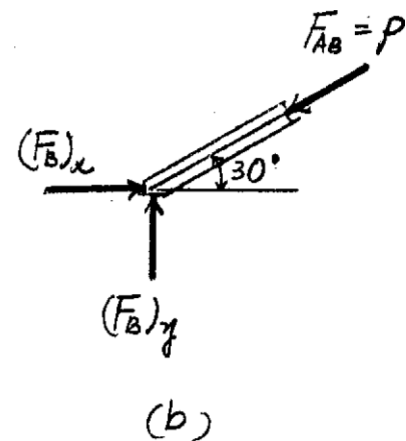
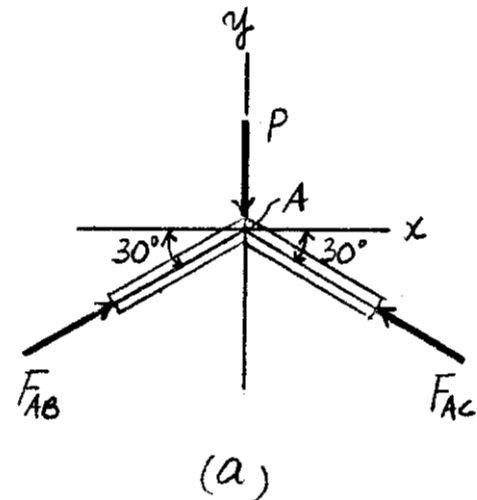
$$\sigma_{allow} = \frac{F_{AB}}{A_{AB}}; \quad 3(10^3) = \frac{P}{3(1.25)}$$

$$P = 11\,250 \text{ lb} = 11.25 \text{ kip}$$

$$\tau_{allow} = \frac{V_{a-a}}{A_{a-a}}; \quad 0.75(10^3) = \frac{0.8660P}{3(3.5)}$$

$$P = 9093.27 \text{ lb} = 9.09 \text{ kip (controls)}$$

Ans.



Ans:
 $P = 9.09 \text{ kip}$

1-91. If the allowable bearing stress for the material under the supports at A and B is $(\sigma_b)_{\text{allow}} = 1.5 \text{ MPa}$, determine the size of *square* bearing plates A' and B' required to support the load. Dimension the plates to the nearest mm. The reactions at the supports are vertical. Take $P = 100 \text{ kN}$.

Referring to the FBD of the beam, Fig. a

$$\zeta + \Sigma M_A = 0; \quad N_B(3) + 40(1.5)(0.75) - 100(4.5) = 0 \quad N_B = 135 \text{ kN}$$

$$\zeta + \Sigma M_B = 0; \quad 40(1.5)(3.75) - 100(1.5) - N_A(3) = 0 \quad N_A = 25.0 \text{ kN}$$

For plate A' ,

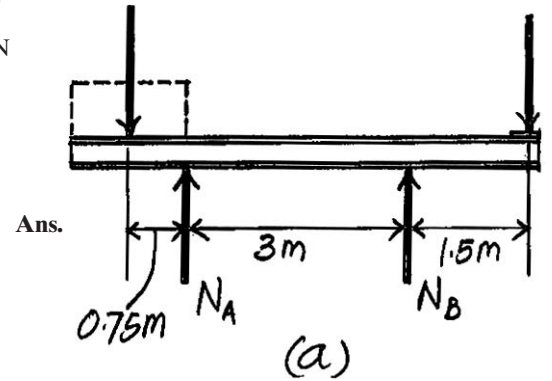
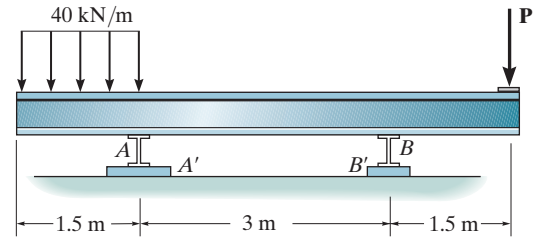
$$(\sigma_b)_{\text{allow}} = \frac{N_A}{A_{A'}}; \quad 1.5(10^6) = \frac{25.0(10^3)}{a_{A'}^2}$$

$$a_{A'} = 0.1291 \text{ m} = 130 \text{ mm}$$

For plate B' ,

$$\sigma_{\text{allow}} = \frac{N_B}{A_{B'}}; \quad 1.5(10^6) = \frac{135(10^3)}{a_{B'}^2}$$

$$a_{B'} = 0.300 \text{ m} = 300 \text{ mm}$$

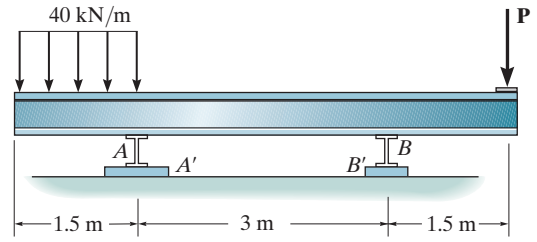


Ans.

Ans:

$$a_{A'} = 130 \text{ mm}, a_{B'} = 300 \text{ mm}$$

*1-92. If the allowable bearing stress for the material under the supports at A and B is $(\sigma_b)_{\text{allow}} = 1.5 \text{ MPa}$, determine the maximum load P that can be applied to the beam. The bearing plates A' and B' have square cross sections of $150 \text{ mm} \times 150 \text{ mm}$ and $250 \text{ mm} \times 250 \text{ mm}$, respectively.



Referring to the FBD of the beam, Fig. a ,

$$\zeta + \sum M_A = 0; \quad N_B(3) + 40(1.5)(0.75) - P(4.5) = 0 \quad N_B = 1.5P - 15$$

$$\zeta + \sum M_B = 0; \quad 40(1.5)(3.75) - P(1.5) - N_A(3) = 0 \quad N_A = 75 - 0.5P$$

For plate A' ,

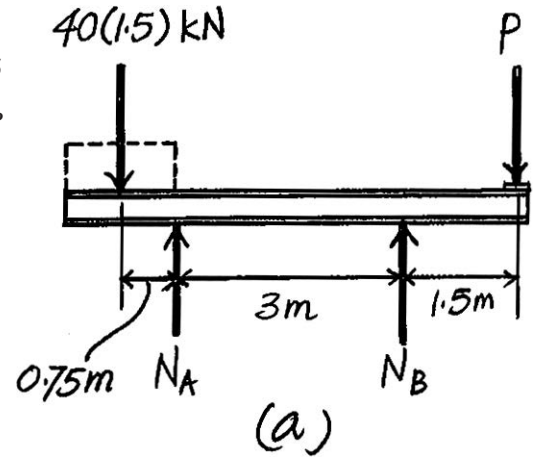
$$(\sigma_b)_{\text{allow}} = \frac{N_A}{A_{A'}}; \quad 1.5(10^6) = \frac{(75 - 0.5P)(10^3)}{0.15(0.15)}$$

$$P = 82.5 \text{ kN}$$

For plate B' ,

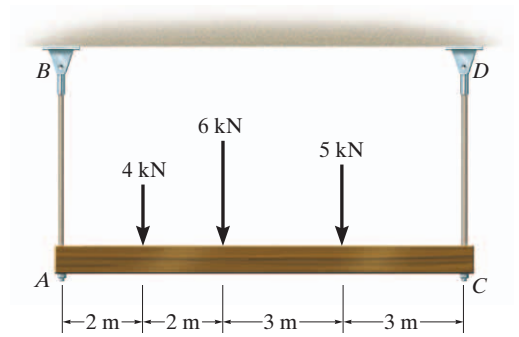
$$(\sigma_b)_{\text{allow}} = \frac{N_B}{A_{B'}}; \quad 1.5(10^6) = \frac{(1.5P - 15)(10^3)}{0.25(0.25)}$$

$$P = 72.5 \text{ kN} \quad (\text{Controls!})$$



Ans.

1-93. The rods AB and CD are made of steel. Determine their smallest diameter so that they can support the dead loads shown. The beam is assumed to be pin connected at A and C . Use the LRFD method, where the resistance factor for steel in tension is $\phi = 0.9$, and the dead load factor is $\gamma_D = 1.4$. The failure stress is $\sigma_{fail} = 345$ MPa.



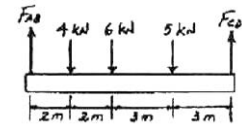
Support Reactions:

$$\zeta + \Sigma M_A = 0; \quad F_{CD}(10) - 5(7) - 6(4) - 4(2) = 0$$

$$F_{CD} = 6.70 \text{ kN}$$

$$\zeta + \Sigma M_C = 0; \quad 4(8) + 6(6) + 5(3) - F_{AB}(10) = 0$$

$$F_{AB} = 8.30 \text{ kN}$$



Factored Loads:

$$F_{CD} = 1.4(6.70) = 9.38 \text{ kN}$$

$$F_{AB} = 1.4(8.30) = 11.62 \text{ kN}$$

For rod AB

$$0.9[345(10^6)] \pi \left(\frac{d_{AB}}{2} \right)^2 = 11.62(10^3)$$

$$d_{AB} = 0.00690 \text{ m} = 6.90 \text{ mm}$$

Ans.

For rod CD

$$0.9[345(10^6)] \pi \left(\frac{d_{CD}}{2} \right)^2 = 9.38(10^3)$$

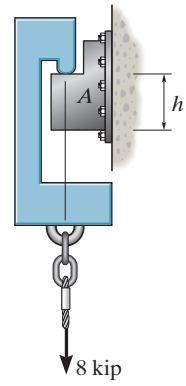
$$d_{CD} = 0.00620 \text{ m} = 6.20 \text{ mm}$$

Ans.

Ans:

$$d_{AB} = 6.90 \text{ mm}, d_{CD} = 6.20 \text{ mm}$$

1-94. The aluminum bracket A is used to support the centrally applied load of 8 kip. If it has a constant thickness of 0.5 in., determine the smallest height h in order to prevent a shear failure. The failure shear stress is $\tau_{\text{fail}} = 23$ ksi. Use a factor of safety for shear of $\text{F.S.} = 2.5$.



Equation of Equilibrium:

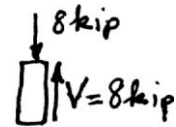
$$+\uparrow \Sigma F_y = 0; \quad V - 8 = 0 \quad V = 8.00 \text{ kip}$$

Allowable Shear Stress: Design of the support size

$$\tau_{\text{allow}} = \frac{\tau_{\text{fail}}}{\text{F.S.}} = \frac{V}{A}; \quad \frac{23(10^3)}{2.5} = \frac{8.00(10^3)}{h(0.5)}$$

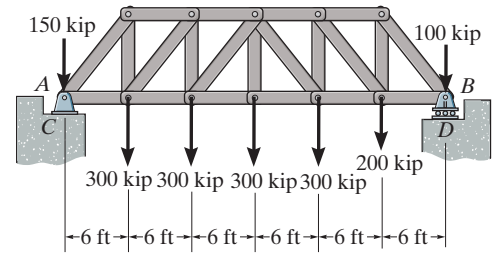
$$h = 1.74 \text{ in.}$$

Ans.



Ans:
 $h = 1.74 \text{ in.}$

1-95. The pin support A and roller support B of the bridge truss are supported on concrete abutments. If the bearing failure stress of the concrete is $(\sigma_{\text{fail}})_b = 4$ ksi, determine the required minimum dimension of the square bearing plates at C and D to the nearest $\frac{1}{16}$ in. Apply a factor of safety of 2 against failure.



Internal Loadings: The forces acting on the bearing plates C and D can be determined by considering the equilibrium of the free-body diagram of the truss shown in Fig. a ,

$$\zeta + \Sigma M_A = 0; \quad B_y(36) - 100(36) - 200(30) - 300(24) - 300(18) - 300(12) - 300(6) = 0$$

$$B_y = 766.67 \text{ kip}$$

$$\zeta + \Sigma M_B = 0; \quad 150(36) + 300(30) + 300(24) + 300(18) + 300(12) + 200(6) - A_y(36) = 0$$

$$A_y = 883.33 \text{ kip}$$

Thus, the axial forces acting on C and D are

$$F_C = A_y = 883.33 \text{ kip} \qquad F_D = B_y = 766.67 \text{ kip}$$

Allowable Bearing Stress:

$$(\sigma_{\text{allow}})_b = \frac{(\sigma_{\text{fail}})_b}{\text{F.S.}} = \frac{4}{2} = 2 \text{ ksi}$$

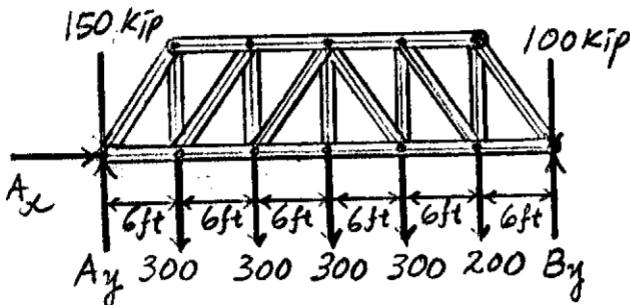
Using this result,

$$(\sigma_{\text{allow}})_b = \frac{F_D}{A_D}; \quad 2(10^3) = \frac{766.67(10^3)}{a_D^2}$$

$$a_D = 19.58 \text{ in.} = 19\frac{5}{8} \text{ in.} \qquad \text{Ans.}$$

$$(\sigma_{\text{allow}})_b = \frac{F_C}{A_C}; \quad 2(10^3) = \frac{883.33(10^3)}{a_C^2}$$

$$a_C = 21.02 \text{ in.} = 21\frac{1}{16} \text{ in.} \qquad \text{Ans.}$$

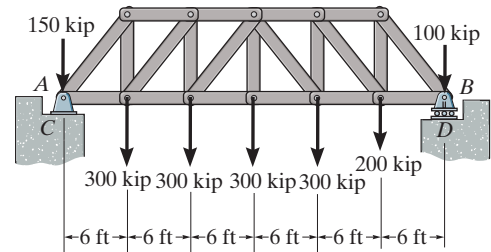


(a)

Ans:

Use $a_D = 19\frac{5}{8}$ in., $a_C = 21\frac{1}{16}$ in.

*1-96. The pin support A and roller support B of the bridge truss are supported on the concrete abutments. If the square bearing plates at C and D are 21 in. \times 21 in., and the bearing failure stress for concrete is $(\sigma_{fail})_b = 4$ ksi, determine the factor of safety against bearing failure for the concrete under each plate.



Internal Loadings: The forces acting on the bearing plates C and D can be determined by considering the equilibrium of the free-body diagram of the truss shown in Fig. a ,

$$\zeta + \Sigma M_A = 0; B_y(36) - 100(36) - 200(30) - 300(24) - 300(18) - 300(12) - 300(6) = 0$$

$$B_y = 766.67 \text{ kips}$$

$$\zeta + \Sigma M_B = 0; 150(36) + 300(30) + 300(24) + 300(18) + 300(12) + 200(6) - A_y(36) = 0$$

$$A_y = 883.33 \text{ kips}$$

Thus, the axial forces acting on C and D are

$$F_C = A_y = 883.33 \text{ kips} \quad F_D = B_y = 766.67 \text{ kips}$$

Allowable Bearing Stress: The bearing area on the concrete abutment is $A_b = 21(21) = 441 \text{ in}^2$. We obtain

$$(\sigma_b)_C = \frac{F_C}{A_b} = \frac{883.33}{441} = 2.003 \text{ ksi}$$

$$(\sigma_b)_D = \frac{F_D}{A_b} = \frac{766.67}{441} = 1.738 \text{ ksi}$$

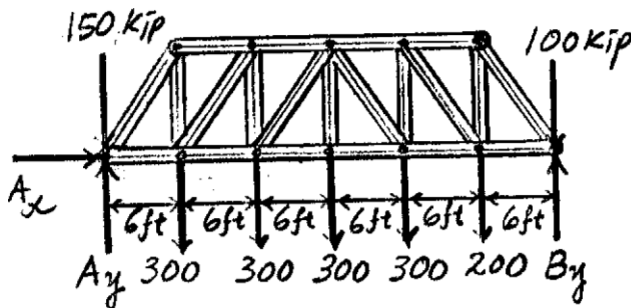
Using these results,

$$(\text{F.S.})_C = \frac{(\sigma_{fail})_b}{(\sigma_b)_C} = \frac{4}{2.003} = 2.00$$

Ans.

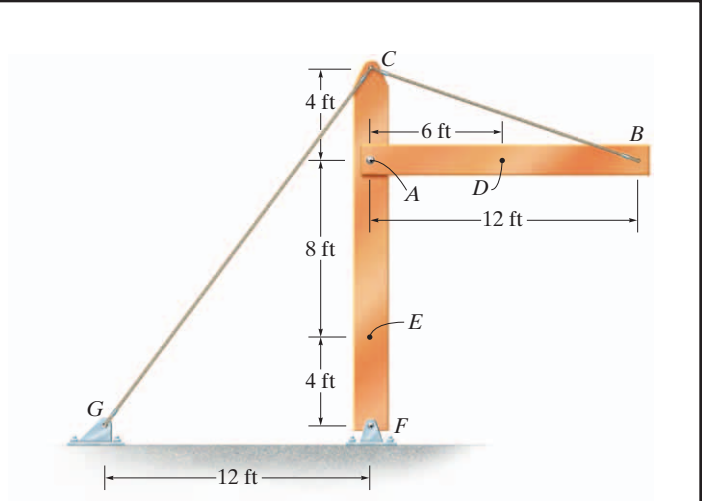
$$(\text{F.S.})_D = \frac{(\sigma_{fail})_b}{(\sigma_b)_D} = \frac{4}{1.738} = 2.30$$

Ans.



(a)

1-97. The beam AB is pin supported at A and supported by a cable BC . A separate cable CG is used to hold up the frame. If AB weighs 120 lb/ft and the column FC has a weight of 180 lb/ft , determine the resultant internal loadings acting on cross sections located at points D and E . Neglect the thickness of both the beam and column in the calculation.



Segment AD :

$$\begin{aligned} \rightarrow \Sigma F_x = 0; \quad N_D + 2.16 &= 0; \quad N_D = -2.16 \text{ kip} \\ +\downarrow \Sigma F_y = 0; \quad V_D + 0.72 - 0.72 &= 0; \quad V_D = 0 \\ \curvearrowright +\Sigma M_D = 0; \quad M_D - 0.72(3) &= 0; \quad M_D = 2.16 \text{ kip} \cdot \text{ft} \end{aligned}$$

Segment FE :

$$\begin{aligned} \leftarrow \Sigma F_x = 0; \quad V_E - 0.54 &= 0; \quad V_E = 0.540 \text{ kip} \\ +\downarrow \Sigma F_y = 0; \quad N_E + 0.72 - 5.04 &= 0; \quad N_E = 4.32 \text{ kip} \\ \curvearrowright +\Sigma M_E = 0; \quad -M_E + 0.54(4) &= 0; \quad M_E = 2.16 \text{ kip} \cdot \text{ft} \end{aligned}$$

Ans.

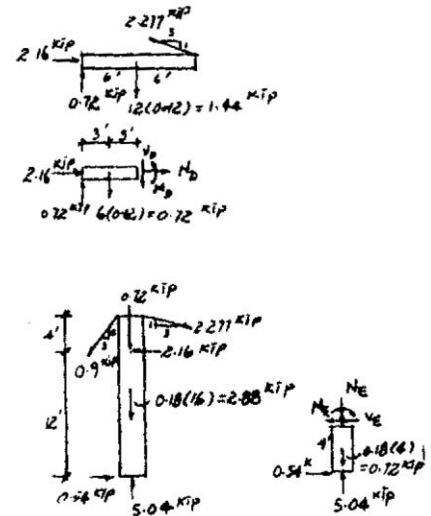
Ans.

Ans.

Ans.

Ans.

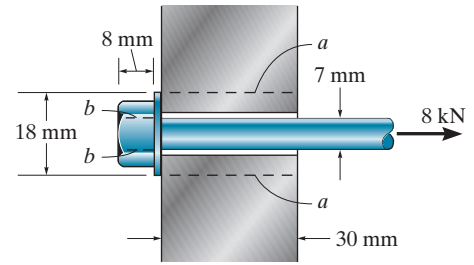
Ans.



Ans:

$$\begin{aligned} N_D = -2.16 \text{ kip}, \quad V_D = 0, \quad M_D = 2.16 \text{ kip} \cdot \text{ft}, \\ V_E = 0.540 \text{ kip}, \quad N_E = 4.32 \text{ kip}, \quad M_E = 2.16 \text{ kip} \cdot \text{ft} \end{aligned}$$

1-98. The long bolt passes through the 30-mm-thick plate. If the force in the bolt shank is 8 kN, determine the average normal stress in the shank, the average shear stress along the cylindrical area of the plate defined by the section lines *a-a*, and the average shear stress in the bolt head along the cylindrical area defined by the section lines *b-b*.



Ans.

$$\sigma_s = \frac{P}{A} = \frac{8 (10^3)}{\frac{\pi}{4} (0.007)^2} = 208 \text{ MPa}$$

Ans.

$$(\tau_{\text{avg}})_a = \frac{V}{A} = \frac{8 (10^3)}{\pi (0.018)(0.030)} = 4.72 \text{ MPa}$$

Ans.

$$(\tau_{\text{avg}})_b = \frac{V}{A} = \frac{8 (10^3)}{\pi (0.007)(0.008)} = 45.5 \text{ MPa}$$

Ans:

$$\sigma_s = 208 \text{ MPa}, (\tau_{\text{avg}})_a = 4.72 \text{ MPa},$$

$$(\tau_{\text{avg}})_b = 45.5 \text{ MPa}$$

1-99. To the nearest $\frac{1}{16}$ in., determine the required thickness of member BC and the diameter of the pins at A and B if the allowable normal stress for member BC is $\sigma_{\text{allow}} = 29$ ksi and the allowable shear stress for the pins is $\tau_{\text{allow}} = 10$ ksi.

Referring to the FBD of member AB , Fig. a ,

$$\zeta + \Sigma M_A = 0; \quad 2(8)(4) - F_{BC} \sin 60^\circ (8) = 0 \quad F_{BC} = 9.238 \text{ kip}$$

$$\rightarrow \Sigma F_x = 0; \quad 9.238 \cos 60^\circ - A_x = 0 \quad A_x = 4.619 \text{ kip}$$

$$+\uparrow \Sigma F_y = 0; \quad 9.238 \sin 60^\circ - 2(8) + A_y = 0 \quad A_y = 8.00 \text{ kip}$$

Thus, the force acting on pin A is

$$F_A = \sqrt{A_x^2 + A_y^2} = \sqrt{4.619^2 + 8.00^2} = 9.238 \text{ kip}$$

Pin A is subjected to single shear, Fig. c , while pin B is subjected to double shear, Fig. b .

$$V_A = F_A = 9.238 \text{ kip} \quad V_B = \frac{F_{BC}}{2} = \frac{9.238}{2} = 4.619 \text{ kip}$$

For member BC

$$\sigma_{\text{allow}} = \frac{F_{BC}}{A_{BC}}; \quad 29 = \frac{9.238}{1.5(t)} \quad t = 0.2124 \text{ in.}$$

$$\text{Use } t = \frac{1}{4} \text{ in.}$$

Ans.

For pin A ,

$$\tau_{\text{allow}} = \frac{V_A}{A_A}; \quad 10 = \frac{9.238}{\frac{\pi}{4} d_A^2} \quad d_A = 1.085 \text{ in.}$$

$$\text{Use } d_A = 1\frac{1}{8} \text{ in.}$$

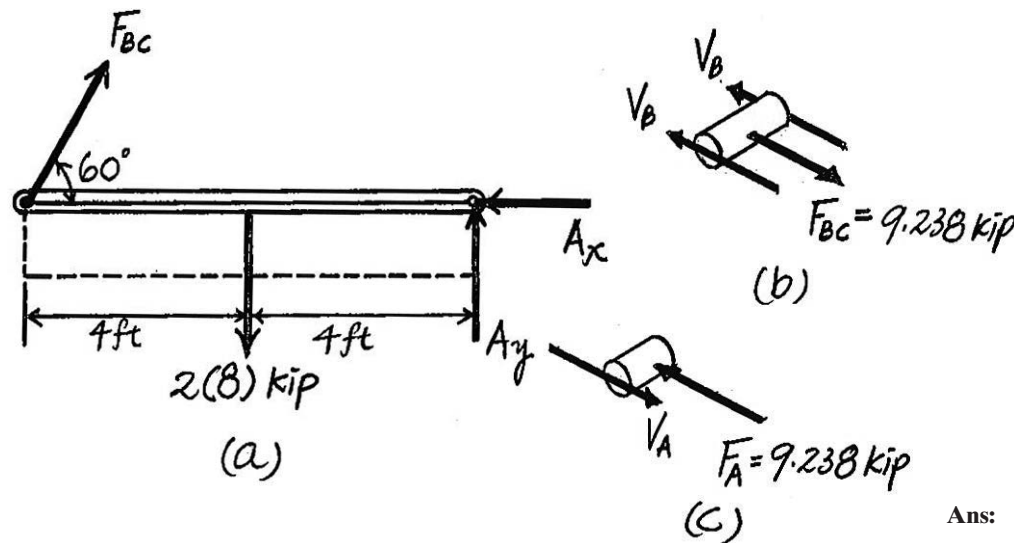
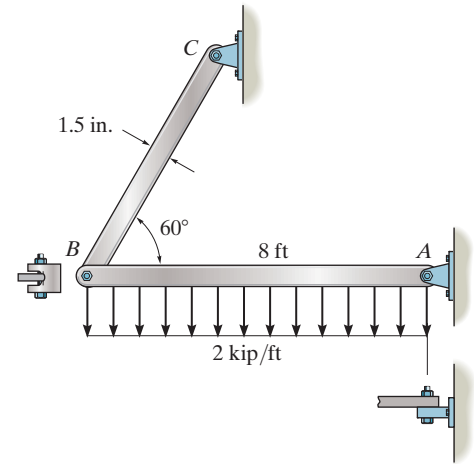
Ans.

For pin B ,

$$\tau_{\text{allow}} = \frac{V_B}{A_B}; \quad 10 = \frac{4.619}{\frac{\pi}{4} d_B^2} \quad d_B = 0.7669 \text{ in.}$$

$$\text{Use } d_B = \frac{13}{16} \text{ in.}$$

Ans.



Ans:
Use $t = \frac{1}{4}$ in., $d_A = 1\frac{1}{8}$ in., $d_B = \frac{13}{16}$ in.

***1-100.** The circular punch B exerts a force of 2 kN on the top of the plate A . Determine the average shear stress in the plate due to this loading.

Average Shear Stress: The shear area $A = \pi(0.004)(0.002) = 8.00(10^{-6})\pi \text{ m}^2$

$$\tau_{\text{avg}} = \frac{V}{A} = \frac{2(10^3)}{8.00(10^{-6})\pi} = 79.6 \text{ MPa}$$

Ans.

